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Modern electrodynamics for the Earth's physics

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Electromagnetic methods for studying the Earth, electromagnetic techniques of geophysical prospecting, in particular, are conventionally based on the fact that any electromagnetic field, either observed on the Earth or artificially excited by generators, satisfy Maxwell's equations and is analytically described by these equations.

This opinion is so popular among geophysicists that any minor deviation from the corollaries following Maxwell's equations is considered to be absent in nature. As an example we can mention a non-potential magnetic field [1, 2] in the practically non-conducting Earth's atmosphere, which is excited by temporal magnetic variations of a natural electromagnetic field; an electric field, vertically directed towards the Earth's surface with almost fully absent vertical currents in the atmosphere [3]; the presence of a toroidal magnetic field in the atmosphere also with almost absent the conductivity current through the atmosphere [4]; the presence of ineradicable and sufficiently large errors in the data interpretation in MTZ and ZCB.

In this case, it would appear reasonable to assume that in nature to be exact in its part that generates electromagnetic fields with the sources on the spherical surfaces in the ionosphere or in the Earth's spherical layers, as an example, there could exist an electromagnetic field (not contained in Maxwell's equations!) that is certainly present in experimental data but are not analytically described by Maxwell's equations.

Poloidal and toroidal magnetic fields have been known since the time of Lamb, Love, Backus and others [5–7]. In these publications, a magnetic field consists of the two parts — toroidal and poloidal magnetic fields:

$$\boldsymbol{H} = \boldsymbol{H}_T + \boldsymbol{H}_P. \tag{1}$$

The above-mentioned fields are analytically introduced in the following way

$$\boldsymbol{H}_T = \operatorname{rot}(Q\boldsymbol{r}); \qquad \boldsymbol{H}_P = \operatorname{rot}\operatorname{rot}(Q\boldsymbol{r}). \tag{2}$$

Here $Q = Q(r, \theta, \varphi)$ is an arbitrary mathematical scalar function of three spherical variables of the class C^∞.

From definition (2) it follows the equation, conventionally called dynamo excitation:

$$\operatorname{rot}\boldsymbol{H}_T = \boldsymbol{H}_P. \tag{3}$$

Actually,

$$\operatorname{rot}\boldsymbol{H}_T = \operatorname{rot}\operatorname{rot}(Q\boldsymbol{r}) = \boldsymbol{H}_P. \tag{4}$$

Formula (4) shows at first that, in the spherical fields, vortices of the non-Maxwell toroidal non-force magnetic field $\boldsymbol{H}_T$ [8] generate a force poloidal magnetic field $\boldsymbol{H}_P$ but not the density of the conductivity electric current as it is according to the first Maxwell's equation. Secondly, dimensions of the toroidal magnetic field $\boldsymbol{H}_T$ and of its rotor are the same in [Gs] only because the rotor of the toroidal magnetic field excites a different Maxwell's force poloidal field (4).

Equation (3), its dimension to be exact or the correct understanding of its physical essence, is underestimated by some well-known scientists [6, 9], who artificially prescribed different dimensions to one and the same function Q, reasoning from the general form of formulas (2). It should be noted again that the function Q is a mathematical scalar dimensionless function of the class C^∞. Initially, only physical values, i.e., electromagnetic fields and electromagnetic constants, have a dimension.

Further, let us transform formula (4) so that instead of the function Q other and different arbitrary functions be on its right- and left-hand sides. Since differential operators are a part of (4), we can add to its right- and left-hand sides different constants:

$$\operatorname{rot}\boldsymbol{H}_T = \operatorname{rot}\operatorname{rot}\big((Q+C_1)\boldsymbol{r}\big) = \boldsymbol{H}_P = \operatorname{rot}\operatorname{rot}\big((Q+C_2)\boldsymbol{r}\big) \tag{5}$$

with the notation $Q + C_1 = T(r, \theta, \varphi)$, $Q + C_2 = P(r, \theta, \varphi)$, and obtain

$$\operatorname{rot}\boldsymbol{H}_T = \operatorname{rot}\operatorname{rot}(T\boldsymbol{r}) = \boldsymbol{H}_P = \operatorname{rot}\operatorname{rot}(P\boldsymbol{r}). \tag{6}$$

From formulas (5) and (6) follows that in these equalities the functions P and T are repositioned due to their arbitrary nature. Also, the functionals $(Q+C_1)r$ and $(Q+C_2)r$, as well as Pr and Tr are completely repositioned only when $C_1 = C_2$ [10]. Definitions (2) and equations (3) do not lose generality with such a repositioning and do not gain new dimensions in the sense that equations (3)–(5) remain valid and the definition of toroidal and poloidal fields and their dimension remain coordinated with formulas (2).

Further we will confirm this important circumstance using the Helmholtz decomposition to prove formula (3) on the one hand, and on the other hand, to, possibly, introduce the toroidal and poloidal fields with the help of relations (2). Now, let us determine the second, closing (3), dynamo excitation equation. To this end, let us calculate the poloidal field rotor:

$$\operatorname{rot}\boldsymbol{H}_P = \operatorname{rot}\operatorname{rot}\operatorname{rot}(Q\boldsymbol{r}) = -\operatorname{rot}(\Delta Q\boldsymbol{r}). \tag{7}$$

In equation (7), the Laplace operator Δ of the scalar function Q is equal to $\Delta Q = -\frac{\gamma}{\eta}Q$ when identifying it with a diffusion potential [11, 12]. Here γ [m/s] is the diffusion rate, η [m^2/s] is magnetic viscosity. Then we obtain

$$\operatorname{rot} \boldsymbol{H}_P = \frac{\gamma}{\eta} \operatorname{rot}(Q\boldsymbol{r}) = \frac{\gamma}{\eta} \boldsymbol{H}_T. \tag{8}$$

Equations (3) and (8) are a closed pair for dynamo excitation of the magnetic field:

$$\operatorname{rot} \boldsymbol{H}_T = \boldsymbol{H}_P; \qquad \operatorname{rot} \boldsymbol{H}_P = \frac{\gamma}{\eta} \boldsymbol{H}_T. \tag{9}$$

A pair of equations (9) allows the evaluation (with prescribed γ and η) of levels of stress of magnetic fields at which a self-supporting dynamo excitation of a magnetic field with a minimum "primer" appears possible [4]. The three well-known anti-dynamo theorems are not applicable for the spherical sources and the sources on spherical surfaces [13–15].

All the above-mentioned results can be found in the Helmholtz theorem about rotor and divergence of the vector-function, presented by the three scalar arbitrary functions $P(r, \theta, \varphi)$, $T(r, \theta, \varphi)$, $F(r, \theta, \varphi)$ of the class C^∞. The Helmholtz theorem in spherical domains is of the form:

$$\boldsymbol{H} = \operatorname{grad} F + \operatorname{rot}(T\boldsymbol{r}) + \operatorname{rot}\operatorname{rot}(P\boldsymbol{r}). \tag{10}$$

Here, as earlier

$$\boldsymbol{H}_T = \operatorname{rot}(T\boldsymbol{r}), \qquad \boldsymbol{H}_P = \operatorname{rot}\operatorname{rot}(P\boldsymbol{r}), \qquad \boldsymbol{H}_M = \operatorname{grad} F. \tag{11}$$

Each term in (10) is of dimension [Gs] because the original field $\boldsymbol{H}$ is in [Gs].

The uniqueness of decomposition (10) is ensured by the theorem of uniqueness that holds that for the uniqueness of decomposition (10) the presence of the field component, $\boldsymbol{H}$ in our case, at all the points of the spherical surface S, covering the vector field $\boldsymbol{H}$, normal to this surface $\boldsymbol{H}_{Pr}(r, \theta, \varphi)$, is required.

Let us reason that the potential part of the original field equal to $\boldsymbol{H}_M = \operatorname{grad} \boldsymbol{F}$ be absent. Let us consider only the part without divergence of formula (10). For this remaining part we will set a problem of defining the functions T, P by the original fields $\boldsymbol{H}_T$, $\boldsymbol{H}_P$. To this end it is necessary to carry out the following transformations:

$$\begin{aligned}
(\boldsymbol{r} \cdot \boldsymbol{H}_P) &= \boldsymbol{r} \cdot \operatorname{rot}\operatorname{rot}(P\boldsymbol{r}) = \boldsymbol{r} \cdot \left[\nabla\nabla \cdot (P\boldsymbol{r}) - \nabla^2 (P\boldsymbol{r})\right] \\
&= \boldsymbol{r} \cdot \{\nabla[\boldsymbol{r} \cdot \nabla P + 3P] - 2\nabla P - \boldsymbol{r}\nabla^2 P\} \\
&= -r^2 \nabla^2 P + \boldsymbol{r} \cdot \nabla(\boldsymbol{r} \cdot \nabla P) + \boldsymbol{r} \cdot \nabla P \\
&= -r^2 \nabla^2 P + \frac{\partial}{\partial r}\left(r^2 \frac{\partial P}{\partial r}\right) = -DP, \\
(\boldsymbol{r} \cdot \operatorname{rot} \boldsymbol{H}_T) &= \boldsymbol{r} \cdot \operatorname{rot}\operatorname{rot}(T\boldsymbol{r}) = -DT.
\end{aligned} \tag{12}$$

Since the direct operator D, according to (12), has no radial derivatives, an inverse operator D^{-1} can be constructed for it and then the functions T and P can be expressed through the original field in the following way:

$$P = -D^{-1}(\boldsymbol{r} \cdot \boldsymbol{H}_P); \qquad T = -D^{-1}(\boldsymbol{r} \cdot \operatorname{rot} \boldsymbol{H}_T). \tag{13}$$

The inverse operator D^{-1} on spherical surfaces can be constructed as follows.

$$\Delta = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{r^2} D. \tag{14}$$

Formula (14) allows to introduce the operator

$$D = \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{15}$$

The inverse operator to the operator D is constructed in the following way. Let $D\psi = f$. Here ψ and f are arbitrary functions of the class C^∞ satisfying zero mean over the spheres:

$$\int_0^{2\pi}\int_0^{\pi} \psi \sin\theta \, d\theta \, d\varphi = \int_0^{2\pi}\int_0^{\pi} f \sin\theta \, d\theta \, d\varphi = 0. \tag{16}$$

Denote

$$S_n(\theta,\varphi) = \sum_{m=0}^{n} A_n^m P_n^m(\cos\theta) e^{im\varphi} \tag{17}$$

with some complex numbers A_n^m.

Then

$$DS_n(\theta,\varphi) = -n(n+1)S_n(\theta,\varphi). \tag{18}$$

Let the functions ψ and f be presented as their decompositions in the spherical functions:

$$\psi = \sum_{n=1}^{\infty} \psi_n(r) S_n(\theta,\varphi); \qquad f = \sum_{n=1}^{\infty} f_n(r) S_n(\theta,\varphi). \tag{19}$$

With allowance for (19), the direct operator is of the form

$$D\psi = -\sum_{n=1}^{\infty} \psi_n(r) n(n+1) S_n(\theta,\varphi) = \sum_{n=1}^{\infty} f_n(r) S_n(\theta,\varphi). \tag{20}$$

Making use of the absolute convergence of expansions series, let us equate the common expansion terms and divide them into $n(n+1)$. As a result we obtain

$$\psi_n(r) S_n(\theta,\varphi) = -f_n(r)\frac{S_n(\theta,\varphi)}{n(n+1)}. \tag{21}$$

Then, having summed all the harmonics, we arrive at

$$\psi = -D^{-1} f = -\sum_{n=1}^{\infty} f_n(r) \frac{S_n(\theta,\varphi)}{n(n+1)}. \tag{22}$$

Formula (22) is the definition of the inverse operator D^{-1} on spherical surfaces. Let us apply the inverse operator D^{-1} to formulas (13). As a result we obtain

$$P = -\sum_{n=1}^{\infty} \boldsymbol{H}_{Pr}(r)\frac{S_n(\theta,\varphi)}{n(n+1)}; \qquad T = -\sum_{n=1}^{\infty} (\operatorname{rot} \boldsymbol{H}_T)(r)\frac{S_n(\theta,\varphi)}{n(n+1)}. \tag{23}$$

If we apply to (23) condition (3) following from (2), i.e., $\operatorname{rot} \boldsymbol{H}_T = \boldsymbol{H}_P$, then from formulas (23) we obtain

$$P = -\sum_{n=1}^{\infty} \boldsymbol{H}_{Pr}(r)\frac{S_n(\theta,\varphi)}{n(n+1)}; \qquad T = -\sum_{n=1}^{\infty} \boldsymbol{H}_{Pr}(r)\frac{S_n(\theta,\varphi)}{n(n+1)}. \tag{24}$$

Thus, the functions P and T are expressed through a radial component of the initial poloidal field $\boldsymbol{H}_{Pr}$ ($\boldsymbol{H}_{Tr} \equiv 0$ by definition). These functions are equal to one another, and this strictly corresponds to the theorem of uniqueness of expansion (10), which for the uniqueness demands the existence of a radial component of the poloidal magnetic field on spherical surfaces, covering the original magnetic field.

Thus, this is the conclusion obtained that closes the empirical input of toroidal and poloidal fields with the one function $Q(r,\theta,\varphi)$ into (1) and (2) and proves the correctness of relation (3) that also uniquely follows from the uniqueness theorem for expansions (10).

If formulas (5) are supplemented with gradients of some other functions for obtaining different functions T and P from one function Q, then one has to introduce the Coulomb graduation of the vector potential $A = (T \cdot \boldsymbol{r}) + \operatorname{rot}(P\boldsymbol{r})$. This version is considered in detail in [10], where one can find the same final result as that in this paper. This circumstance allows us without loss of generality to write down the Helmholtz expansion for the vector field $\boldsymbol{H}$ as follows

$$\boldsymbol{H} = \operatorname{grad} F + \operatorname{rot}(Q\boldsymbol{r}) + \operatorname{rot}\operatorname{rot}(Q\boldsymbol{r}). \tag{25}$$

Formula (25) essentially closes up with definitions from (1) and (2).

Now, there is a good reason to connect the inductive excitation, reflected in Maxwell's equations and the dynamo excitation for obtaining the generalized Maxwell's equations for spherical domains, spherical sources and sources located on spheres:

$$\begin{gathered} \operatorname{rot} \boldsymbol{H}_P = \sigma E_T + \frac{\gamma}{\eta}\boldsymbol{H}_T; \qquad \operatorname{rot} \boldsymbol{H}_T = \boldsymbol{H}_P; \\ \operatorname{div}(\boldsymbol{H}_T, \boldsymbol{H}_P) \equiv 0. \end{gathered} \tag{26}$$

Here E_T is a toroidal electric field.

Boundary conditions for the magnetic fields $\boldsymbol{H}_T$ and $\boldsymbol{H}_P$ are standard, not differing from the well-known. For the function Q they are derived from the boundary conditions for magnetic fields and are of the form

$$Q\Big|_{r=R_0} \neq 0; \qquad \frac{1}{\sin\theta}\frac{\partial Q}{\partial\varphi}\Big|_{r=R_0} \neq 0; \qquad \frac{\partial Q}{\partial\theta}\Big|_{r=R_0} \neq 0. \tag{27}$$

However, the potentiality principle of the magnetic field, in the Earth's atmosphere, for example, is formulated quite in a different manner. In fact,

$$\oint(\boldsymbol{H}_P \cdot \boldsymbol{dl}) = \int(\operatorname{rot}\boldsymbol{H}_P \cdot \boldsymbol{ds}) = \int J_n ds\Big|_{J_n=0} = 0. \tag{28}$$

From (28) follows that a poloidal field in the atmosphere, where $J_n = 0$, is a potential field $\operatorname{rot}\boldsymbol{H}_P = 0$. While a toroidal field being trivially not potential anywhere because of $\operatorname{rot}\boldsymbol{H}_T = \boldsymbol{H}_P$ can exist in the Earth's atmosphere due to the presence in it a poloidal magnetic field. In fact,

$$\oint(\boldsymbol{H}_T \cdot \boldsymbol{dl}) = \int(\operatorname{rot}\boldsymbol{H}_T \cdot \boldsymbol{ds}) = \int H_{Pr} ds\Big|_{\boldsymbol{H}_{Pr}\neq 0} \neq 0. \tag{29}$$

With the use of relation (29), a problem that has long been known [1,2] is solved. In the above-cited publications, magnetic fields without potential of the Earth's field variations in the Earth's atmosphere were found. From (29) it follows that the part without potential of the magnetic field in the atmosphere is nothing but a toroidal magnetic field, whose vortices do not generate an electric current through the atmosphere. A toroidal magnetic field appears in the Earth's atmosphere through boundary conditions (27).

The presence of the toroidal magnetic field in the atmosphere according to (29) breaks down the known from literature myth about its screening with the Earth's mantle [4].

An equally important disproof follows from formula (29). It concerns an existing opinion about the unimodular feature of the electromagnetic field in MTZ and ZCB. The effect of the spherical property of the Earth's surface and the ionosphere brings about — due to "spreading" of the source of the field — the appearance of the second modification in observed electromagnetic fields, to be exact, of the mode $H_{T\theta}$, $H_{T\varphi}$, $E_{P\theta}$, $E_{P\varphi}$, E_{Pr} along with the major mode $H_{P\theta}$, $H_{P\varphi}$, H_{Pr}, $E_{T\theta}$, $E_{T\varphi}$ [4].

Neglect of this factor results in appearance of irremovable errors when interpreting the data in MTZ and ZSB. The presence of E_{Pr} (E_z) in the air, measured and described in [3], is also due to the appearance in the atmosphere of the second modification of the field because of the spherical effect.

Over prolonged periods neither in laboratory conditions nor in nature the sources of the toroidal magnetic field could not be detected. The obstacle was the above-mentioned so-called anti-dynamo theorems [6,13,14]. The monograph [16] describes a laboratory source of the toroidal magnetic field, called a two-disk dynamo. In literature, until now one can find references to this approach to creating a toroidal field. However, a natural source was found in [4, 8], where it was shown that electric conductivity currents located in spherical domains of the Earth or on the spherical surfaces in the ionosphere necessarily generate a non-force toroidal electromagnetic field in addition to a force poloidal magnetic field [8], where this fact is analytically verified based on the equation of the full current, following from the generalized Maxwell's equations (26): $\Delta\boldsymbol{A} = \boldsymbol{J}^p$, $\boldsymbol{J}^p = \sigma\boldsymbol{E}_T + \frac{\gamma}{\eta}\boldsymbol{H}_T$. In this case $\frac{\gamma}{\eta}\boldsymbol{H}_T = \sigma(\gamma\mu\boldsymbol{H}_T) = \sigma(\boldsymbol{E}_T[v/m])$ is that supplementary current, which generates a toroidal magnetic field. In fact, the projection of the equation $\Delta\boldsymbol{A} = \boldsymbol{J}^p$ onto the axis of the spherical coordinate system is of the form

$$\Delta_\theta A = \frac{\partial^2 A_\theta}{\partial r^2} + \frac{2\partial A_\theta}{r\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial^2 A_\theta}{\partial\varphi^2} + \frac{1}{r^2}\frac{\partial^2 A_\theta}{\partial\theta^2} - \frac{1}{r^2}\frac{\cos\theta}{\sin\theta}\frac{\partial A_\theta}{\partial\theta} - \frac{A_\theta}{r^2\sin^2\theta} - 2\frac{\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\varphi}{\partial\varphi} + \frac{2}{r}\frac{\partial Q}{\partial\theta} = J^p_\theta, \tag{30}$$

$$\Delta_\varphi A = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial A_\varphi}{\partial\varphi} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 A_\varphi}{\partial\varphi^2} + \frac{1}{r}\frac{\partial^2 rA_\varphi}{\partial r^2} + \frac{\cos\theta}{r^2\sin^2\theta}\frac{\partial A_\theta}{\partial\varphi} - \frac{\cos\theta}{r^2\sin\theta}\frac{\partial A_\varphi}{\partial\varphi} + \frac{1}{r^2}\frac{\partial}{\partial\theta}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta A_\varphi - \frac{1}{r}\frac{\partial^2 A_\varphi}{\partial\theta\partial\varphi} + \frac{2}{r\sin\theta}\frac{\partial Q}{\partial\varphi} = J^p_\varphi.$$

It is easy to note that, in (30), formulas

$$\frac{2}{r}\frac{\partial Q}{\partial\theta} = -\frac{2}{r}\boldsymbol{H}_{T\varphi}, \qquad \frac{2}{r\sin\theta}\frac{\partial Q}{\partial\varphi} = \frac{2}{r}\boldsymbol{H}_{T\theta} \tag{31}$$

are $\frac{2}{r}$-multiplied components of the toroidal magnetic field excited by J^P_θ and J^p_φ components of the full spherical current located either in spherical layers or on the spherical surfaces. Therefore, for the existence in the atmosphere of a toroidal magnetic field the presence of natural spherical components J^p_θ and J^p_φ of the electric field is sufficient. There is no need in the radial electric current for excitation of a toroidal magnetic field in the air. The toroidal magnetic field from J^p_θ- and J^p_φ-components of the current freely penetrates into non-conducting regions including the Earth's atmosphere with the help of boundary conditions (27), not creating in them an additional electric current by virtue of formula (3).

Thus, a general description of constant magnetic fields from spherical sources leads to the generalized Maxwell's equations (26) giving the consequences verified by an experiment in terrain. Such experiments are of constant, observable character on the world-wide net of magnetic observatories. The data obtained on this net are subject to interpretation, to be done, in our opinion, on the basis of the general system of equations (26). The results of such an interpretation can be found in the monograph [4].

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