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Towards decidability of timed testing*

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In the paper, we construct a formula that characterizes a timed event structure up to the timed *must*-preorder.

1. Introduction

An important component of every process theory is a notion of equivalence between processes. Typically, equivalences are used in the setting of specification and verification both to compare two distinct systems and to reduce the structure of a system. Over the past several years, a variety of equivalences have been proposed, and the relationship between them has been quite well-understood (see, for example, [9]).

Among the major equivalences are testing ones presented in [8]. Two processes are considered to be testing equivalent, if there is no test that can distinguish them. A test itself is usually a process applied to another process by computing them together in parallel. A particular computation is considered to be successful, if the test reaches a designated successful state, and the process passes the test if every computation is successful. This notion is intuitively appealing; it has led to a well-developed mathematical theory of processes that ties together the equivalences and preorders. However, no characterization of these equivalences has led to an algorithmic solution for finite-state processes. Therefore, testing decision procedures are based on reduction of testing to bisimulation [6]. These equivalences have been considered for formal system models without time delays [1, 6, 8, 10].

Recently, testing equivalences have been developed for models with time. One of the papers [13] devoted to this subject investigates different betting semantics of "*must*" win and "*may*" win, taken from the testing methodology, in the context of an event structure model with delayed actions. Papers [7] and [14] have treated timed testing for discrete and dense time transition models, respectively. The latter paper also tries to provide a testing decision procedure that uses the untimed bisimulation between deterministic graphs built from mutually refined timer region graphs that are a finite abstraction of the operational semantics of the model under consideration.

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In paper [4], a framework for testing preorders and equivalences in the setting of timed event structures has been developed. But as for the characterization and the decision procedure, it turns out that the results of [14] were not the case for some timed event structures. Since these timed event structures can be easily transformed to timed transition systems, it seems that the results of [14] are valid only for some subclass of timed transition systems. So, we try to give the alternative characterization of the timed testing relations. Moreover, we have found a subclass of structures in which we could reduce timed testing relations to the corresponding variants of symbolic bisimulations.

This paper is devoted to decidability of timed *must*-equivalences for timed event structures. We try to reduce this problem to the model-checking one. As a basic logic, we take the timed logic L_ν. This logic has been defined in [12] and used for construction of a characteristic formula for a timed automaton up to the timed bisimilarity and, as a consequence, for reduction of the timed bisimilarity decidability problem to the model-checking one. It is known that the latter problem is decidable.

Here we construct a characteristic formula up to the timed *must*-preoders. We do it only for timed event structures without internal actions, but this approach can be used for those with internal actions, too.

The rest of the paper is organized as follows. In Section 2, we remind the basic notions concerned with timed event structures and timed testing. The timed modal logic L_ν is described in Section 3. In Section 4, we construct a formula which characterizes a timed event structure up to the timed *must*-preoder.

2. Timed event structures

In this section, we introduce a model of timed event structures that is a real time extension of Winskel's model of prime event structures [15] by equipping events with time intervals.

We first recall a notion of an event structure. The main idea behind event structures is to view the distributed computations as action occurrences, called events, together with a notion of causal dependence between events (which are reasonably characterized via a partial order). Moreover, to model nondeterminism, there is a notion of conflicting (mutually incompatible) events. A labelling function determines which action corresponds to an event.

Let Act be a finite set of visible actions and τ be an internal action. Then $Act_\tau = Act \cup \{\tau\}$.

Definition 1. A *(labelled) event structure* over Act_τ is a 4-tuple $S = (E, \leq, \#, l)$, where

- E is a countable set of events;
- $\leq \subseteq E \times E$ is a partial order (the *causality relation*) satisfying the *principle of finite causes*: $\forall e \in E$. $\{e' \in E \mid e' \leq e\}$ is finite;
- $\# \subseteq E \times E$ is a symmetric and irreflexive relation (the *conflict relation*) satisfying the *principle of conflict heredity*: $\forall e, e', e'' \in E$. $e \# e' \leq e'' \Rightarrow e \# e''$;
- $l : E \to Act_\tau$ is a labelling function.

Let $C \subseteq E$. Then C is *left-closed* iff $\forall e, e' \in E$. $e \in C \wedge e' \leq e \Rightarrow e' \in C$; C is *conflict-free* iff $\forall e, e' \in C$. $\neg(e \# e')$; C is a *configuration of* S iff C is left-closed and conflict-free. Let $Conf(S)$ denote the set of all configurations of S. For $C \in Conf(S)$, we define the set of events enabled in C $En(C) = \{e \in E \mid C \cup \{e\} \in Conf(S)\}$.

In the following, we will consider only finite event structures, i.e., the structures whose sets of events are finite.

Before introducing the concept of a timed event structure, we need to propose some auxiliary notations. Let $\mathbf{N}_0$ be the set of natural numbers with zero, $\mathbf{R}^+$ be the set of positive real numbers, and $\mathbf{R}_0^+$ be the set of nonnegative real numbers. For any $d \in \mathbf{R}_0^+$, $\{d\}$ denotes its fractional part, $\lfloor d \rfloor$ and $\lceil d \rceil$ — its smallest and largest integer parts, respectively. Let us define the set $Interv(\mathbf{R}_0^+) = \{(d_1, d_2), (d_1, d_2], [d_1, d_2), [d_1, d_2] \subset \mathbf{R}_0^+ \mid d_1, d_2 \in \mathbf{N}_0\}$.

We are now ready to introduce the concept of timed event structures.

Definition 2. A *(labelled) timed event structure* over Act_τ is a pair $TS = (S, D)$, where

- $S = (E, \leq, \#, l)$ is a (labelled) event structure over Act_τ;
- $D : E \to Interv(\mathbf{R}_0^+)$ is a timing function such that $D(e)$ is a closed interval from $Interv(\mathbf{R}_0^+)$ for all $e \in E$ with $l(e) \in Act$.

In a graphic representation of a timed event structure, the corresponding action labels and time intervals are drawn close to events. If no confusion arises, we will often use action labels instead of the event identifiers to denote events. The $<$-relations are depicted by arcs (omitting those derivable by transitivity), and conflicts are depicted by "#" (omitting those derivable by the conflict heredity). Following these conventions, a trivial example of a labelled timed event structure is shown in Figure 1.

Let $\mathcal{E}_\tau$ denote the set of all labelled timed event structures over Act_τ. For convenience, we fix timed event structures $TS = (S = (E, \leq, \#, l), D)$, $TS' = (S' = (E', \leq', \#', l'), D')$ from the class $\mathcal{E}_\tau$ and work with them further.

$$TS_1 \qquad [0,1]\ a:e_1 \longrightarrow b:e_2\ [0,1]$$
$$\#$$
$$\tau:e_3\ [0,1)$$

Figure 1

A *state* of TS is a pair $M=(C,\delta)$, where $C \in Conf(S)$ and $\delta : E \rightarrow \mathbf{R}_0^+$. The *initial state* of TS is $M_{TS}=(C_0,\delta_0)=(\emptyset,0)$. A state $M=(C,\delta)$ is said to be *terminated*, if $En(C)=\emptyset$. Let $ST(TS)$ denote the set of all states of TS.

A timed event structure progresses through a sequence of states in one of two ways given below.

Let $M_1=(C_1,\delta_1), M_2=(C_2,\delta_2) \in ST(TS)$ such that M_1 is a non-terminated state. An event $e \in En(C_1)$ *may occur* in M_1 (denoted $M_1 \xrightarrow{e}$) if $\delta_1(e) \in D(e)$ and $\forall e' \in En(C_1)\ \exists d \in \mathbf{R}_0^+ \ .\ \delta_1(e')+d \in D(e)$. We write $M_1 \xrightarrow{a}$, if $M_1 \xrightarrow{e}$ and $l(e)=a$. The occurrence of e in M_1 *leads* to M_2 (denoted $M_1 \xrightarrow{e} M_2$), if $M_1 \xrightarrow{e}$, $C_2=C_1 \cup \{e\}$ and

$$\delta_2(e')=\begin{cases} 0, & \text{if } e' \in En(C_2) \setminus En(C_1), \\ \delta_1(e'), & \text{otherwise.} \end{cases}$$

We write $M_1 \xrightarrow{a} M_2$, if $M_1 \xrightarrow{e} M_2$ and $l(e)=a$.
A time $d \in \mathbf{R}^+$ *may pass* in M_1 (denoted $M_1 \xrightarrow{d}$), if $\forall e \in En(C_1)\ \exists d' \in \mathbf{R}_0^+ (d' \geq d)\ .\ \delta_1(e)+d' \in D(e)$. The passage d in M_1 *leads* to M_2 (denoted $M_1 \xrightarrow{d} M_2$), if $C_2=C_1$ and $\delta_2(e)=\delta_1(e)+d$ for all $e \in E$.

The *weak leading relation* $\Rightarrow$ *on states of* TS is the largest relation defined by: $\overset{\epsilon}{\Rightarrow} \Longleftrightarrow \overset{\tau}{\rightarrow}^*$ and $\overset{x}{\Rightarrow} \Longleftrightarrow \overset{\epsilon}{\Rightarrow}\overset{x}{\rightarrow}\overset{\epsilon}{\Rightarrow}$, where $\overset{\tau}{\rightarrow}^*$ is the reflexive and transitive closure of $\overset{\tau}{\rightarrow}$ and $x \in Act \cup \mathbf{R}^+$. We consider the relation $\overset{d}{\Rightarrow}$ as possessing the time continuity property: $M \overset{d_1+d_2}{\Longrightarrow} \Longleftrightarrow M \overset{d_1}{\Rightarrow}\overset{d_2}{\Rightarrow}$ for some $d_1, d_2 \in \mathbf{R}^+$.

From now on, we shall use the following notions and notations. Let $Act(\mathbf{R}_0^+)=\{a(d) \mid a \in Act \wedge d \in \mathbf{R}_0^+\}$ be the set of *timed actions* of Act over $\mathbf{R}_0^+$. Then $(Act(\mathbf{R}_0^+))^*$ is the set of finite *timed words* over $Act(\mathbf{R}_0^+)$. The function $\triangle : (Act(\mathbf{R}_0^+))^* \rightarrow \mathbf{R}_0^+$ measuring the *duration* of a timed word is defined by: $\triangle(\epsilon)=0$, $\triangle(w.a(d))=\triangle(w)+d$. The domain for real-time languages is denoted by $Dom(Act,\ \mathbf{R}_0^+)=\{\langle w,d\rangle \mid w \in (Act(\mathbf{R}_0^+))^*,\ d \in \mathbf{R}_0^+,\ d \geq \triangle(w)\}$. The weak leading relation $\Rightarrow$ is extended to timed words from $(Act(\mathbf{R}_0^+))^*$ and $Dom(Act,\ \mathbf{R}_0^+)$ as follows. Let $d \in \mathbf{R}_0^+$, $d' \in \mathbf{R}^+$, $a \in Act$ and $w \in (Act(\mathbf{R}_0^+))^*$. Then

$$\text{if } M \overset{a}{\Rightarrow} M', \text{ then } M \overset{a(0)}{\Rightarrow} M'; \text{ if } M \overset{d'}{\Rightarrow}\overset{a}{\Rightarrow} M', \text{ then } M \overset{a(d')}{\Rightarrow} M';$$
$$\text{if } M \overset{w}{\Rightarrow}\overset{a(d)}{\Rightarrow} M', \text{ then } M \overset{w.a(d)}{\Longrightarrow} M'; \text{ if } M \overset{w}{\Rightarrow} M', \text{ then } M \overset{\langle w,\ \triangle(w)\rangle}{\Longrightarrow} M';$$
$$\text{if } M \overset{\langle w,d\rangle}{\Longrightarrow}\overset{d'}{\Rightarrow} M', \text{ then } M \overset{\langle w,\ d+d'\rangle}{\Longrightarrow} M'.$$

The set $L(TS) = \{\langle w, d\rangle \in Dom(Act, \mathbf{R}_0^+) \mid M_{TS} \stackrel{\langle w,d\rangle}{\Longrightarrow}\}$ is the *language* of TS. For instance, for the timed event structure TS_1 (see Figure 1) we have $L(TS_1) = \{\langle\epsilon, d_1\rangle, \langle\epsilon, 1\rangle,\ \langle a(d_1), d_1 + d_2\rangle,\ \langle a(1), 1\rangle,\ \langle a(d_1)b(d_2), d_1 + d_2\rangle \mid d_1 + d_2 < 1\}$.

The timed testing relations may be defined in terms of the responses of timed event structures to a collection of tests. We shall, however, use an alternative characterization that relies on the following definitions. Let $M \in ST(TS)$ and $\langle w,\ d\rangle \in Dom(Act,\ \mathbf{R}_0^+)$. Then $S(M) = \{x \in Act_\tau \cup \mathbf{R}^+ \mid M \stackrel{x}{\rightarrow}\}$ and $Acc(TS,\ \langle w,\ d\rangle) = \{S(M') \mid M_{TS} \stackrel{\langle w,\ d\rangle}{\Longrightarrow} M',\ M' \stackrel{\tau}{\not\rightarrow}\}$ (timed acceptance set). Let $N, N' \subset 2^{Act \cup \mathbf{R}^+}$. Then $N \subset\subset N' \iff \forall S \in N\ \exists S' \in N'\ .\ [(S' \mid_{Act} \subseteq S \mid_{Act})\ \wedge\ (S \mid_{\mathbf{R}^+} = \emptyset \Rightarrow S' \mid_{\mathbf{R}^+} = \emptyset)]$; $N \equiv N' \iff N \subset\subset N'\ \wedge\ N' \subset\subset N$.

Definition 3.

- $TS \leq_{must} TS' \iff \forall\langle w,\ d\rangle \in Dom(Act, \mathbf{R}_0^+)\ .\ Acc(TS', \langle w, d\rangle) \subset\subset Acc(TS, \langle w, d\rangle)$;
- $TS \simeq_{must} TS' \iff TS \leq_{must} TS'$ and $TS' \leq_{must} TS$.

An example of timed *must*-equivalent structures is shown in Figure 2(a). The timed event structures TS_3 and TS_3' shown in Figure 2(b) are not timed *must*-equivalent. Let us consider the timed word $\langle w, d\rangle = \langle a(0.5), 1.5\rangle \in L(TS_3) \cap L(TS_3')$. We have $Acc(TS_3, \langle w, d\rangle) = \{\{b, c\} \cup (0, 1]\}$, $Acc(TS_3', \langle w, d\rangle) = \{\{b, c\} \cup (0, 1], \{c\}\}$, i.e., $\neg(Acc(TS_3', \langle w, d\rangle) \subset\subset Acc(TS_3, \langle w, d\rangle))$.

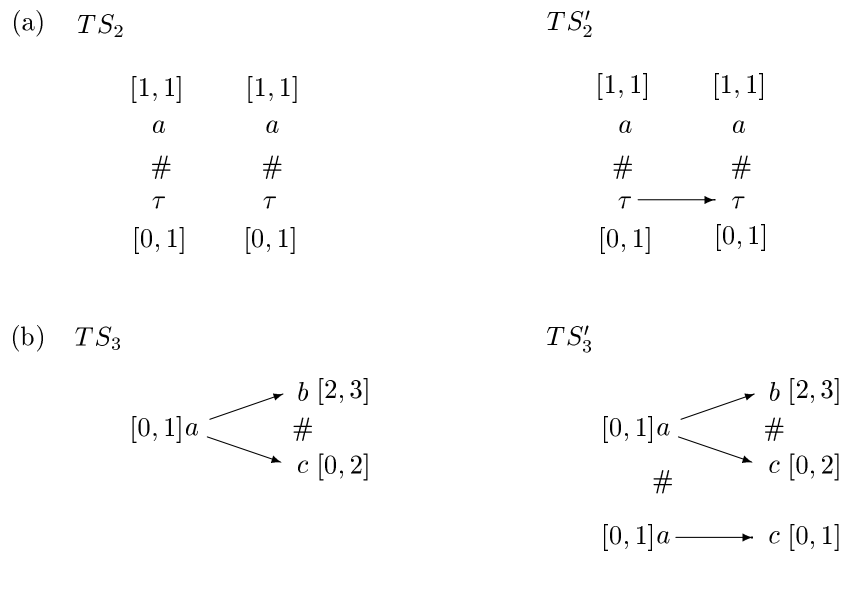

Figure 2

3. Timed modal logic

In [12], a dense-timed logic L_ν was considered. Here we shall recall it and modify a satisfiability relation for timed event structures.

Definition 4. Let K be a finite set of clocks, Id be a set of identifiers and k be an integer. The set of formulas of L_ν over K, Id and k is generated by the abstract syntax with ϕ and ψ ranging over L_ν:

$$\phi := tt \mid ff \mid \phi \wedge \psi \mid \phi \vee \psi \mid \exists\phi \mid \forall\phi \mid \langle a\rangle\phi \mid [a]\phi \mid x \text{ in } \phi \mid x+n \bowtie y+m \mid x \bowtie m \mid Z,$$

where $a \in Act$, $x, y \in K$, $n, m \in \{0, 1, \ldots, k\}$, $\bowtie \in \{=, <, \leq, >, \geq\}$ and $Z \in Id$.

The meaning of the identifiers from Id is specified by a declaration D that assigns a formula of L_ν to each identifier. When D is clear, we write $Z := \phi$ for $D(Z) = \phi$. The K clocks are called formula clocks and a formula ϕ is said to be *closed* if every formula clock x occurs in ϕ in the scope of an "x in ..." operator. Given a timed event structure TS, we interpret the formulas from L_ν over *an extended* state $(C, \delta u)$, where (C, δ) is a state of TS and u is a time assignment for K. Transitions between extended states are defined by: $(C, \delta u) \stackrel{\epsilon(d)}{\to} (C, (\delta + d)(u + d))$ and $(C, \delta u) \stackrel{a}{\to} (C', \delta' u')$ iff $(C, \delta) \stackrel{a}{\to} (C', \delta')$ and $u = u'$. Formally, the satisfaction relation between extended states and formulas is defined as follows:

Definition 5. [1] Let TS be a timed event structure and D be a declaration. The satisfaction relation $\models_D$ is the largest one that satisfies the following implications:

$$\begin{array}{rcl}
(C, \delta u) \models_D tt & \Rightarrow & \text{true}; \\
(C, \delta u) \models_D ff & \Rightarrow & \text{false}; \\
(C, \delta u) \models_D \phi \wedge \psi & \Rightarrow & (C, \delta u) \models_D \phi \text{ and } (C, \delta u) \models_D \psi; \\
(C, \delta u) \models_D \exists\phi & \Rightarrow & \exists d \in R \,.\, (C, \delta + du + d) \models_D \phi; \\
(C, \delta u) \models_D \langle a\rangle\phi & \Rightarrow & \exists (C', \delta') \in ST(TS) \,.\, (C, \delta) \stackrel{a}{\to} (C', \delta') \\
& & \text{and } (C', \delta' u) \models_D \phi; \\
(C, \delta u) \models_D x + m \bowtie y + n & \Rightarrow & u(x) + m \bowtie u(y) + n; \\
(C, \delta u) \models_D x \text{ in } \phi & \Rightarrow & (C, \delta u') \models_D \phi, \text{ where } u' = [\{x\} \to 0]u; \\
(C, \delta u) \models_D Z & \Rightarrow & (C, \delta u) \models_D D(Z).
\end{array}$$

Any relation that satisfies the above implications is called a satisfiability relation. We say that TS *satisfies* a closed formula ϕ from L_ν and write $TS \models \phi$ when $(C_0, \delta_0 u) \models_D \phi$ for any u. Note that if ϕ is closed, then $(C, \delta u) \models_D \phi$ iff $(C, \delta u') \models_D \phi$ for any $u, u' \in \mathbf{R}_0^{+K}$.

[1] For the complete definition, see [12].

4. Formula construction

Further we restrict our model to timed event structures labelled only over *Act*.

Since a timed event structure is defined over a dense time domain, the number of its states is infinite. In order to get a discrete representation of the state-space of a timed event structure, we use the concept of regions (equivalence classes of states) [2]. But we do not construct regions over states of $ST(TS)$ for the following reasons.

One of the problems we meet when trying to develop an algorithm of decidability of timed testing [4] is existence of several regions which contain states reachable by the same timed word. So, we construct a region over states that unite all states of TS which we get by passing some time word. By doing so, we want to exclude nondeterminism when progressing in a timed event structure from state to state. For this purpose we define the notion of common states of TS.

The other problem is to synchronize actions being executed in two timed event structures. Here we decide it by including counters into regions of one timed event structure in order to restrict states of the second one for which a region formula has to be checked.

Some subset of $ST(TS)$ is called a *common state* of TS, i.e., $\mu \subseteq ST(TS)$ is a common state of TS. We shall sometimes denote μ as $(C_1, \ldots, C_n, \delta_1, \ldots, \delta_n)$, where $(C_i, \delta_i) \in \mu$ $(1 \leq i \leq n)$ and $En(\mu) = \bigcup\{En(C) \mid C \in \mu\}$. The *initial* common state of TS is $\mu_0 = \{M_{TS}\}$. The relation $\xrightarrow{z}$ is modified on common states as follows: $\mu \xrightarrow{z} \mu' = \{(C', \delta') \mid \exists (C, \delta) \in \mu \ . \ (C, \delta) \xrightarrow{z} (C', \delta')\}$, where $z \in Act \cup \mathbf{R}^+$. Let $STC(TS)$ denote the set of all common states reachable from μ_0. The leading relation on common states of $STC(TS)$ is extended to timed words from $Dom(Act, \mathbf{R}_0^+)$ just as on the states of $ST(TS)$.

Then the notion of region is defined analogously to Alur's one. Let $\mu = (C_1, \ldots, C_n, \delta_1, \ldots, \delta_n) \neq \mu' = (C'_1, \ldots, C'_n, \delta'_1, \ldots, \delta'_n)$. Then $\mu \simeq \mu'$ iff there exists renaming $\pi(n) : l \to \pi(n)(l)$, where $l = 1, \ldots n$, such that $(C_1, \ldots, C_n) = (C'_{\pi(n)(1)}, \ldots, C'_{\pi(n)(n)})$ and

(i) $\forall 1 \leq i \leq m \ . \ \lfloor \delta_1 | \ldots | \delta_n(i) \rfloor = \lfloor \delta'_{\pi(n)(1)} | \cdots | \delta'_{\pi(n)(n)}(i) \rfloor$,

(ii) $\forall 1 \leq i, j \leq m$.

- $\{\delta_1 | \ldots | \delta_n(i)\} \leq \{\delta_1 | \ldots | \delta_n(j)\} \iff \{\delta'_{\pi(n)(1)} | \cdots | \delta'_{\pi(n)(n)}(i)\} \leq \{\delta'_{\pi(1)} | \cdots | \delta'_{\pi(n)}(j)\}$,
- $\{\delta_1 | \ldots | \delta_n(i)\} = 0 \iff \{\delta'_{\pi(n)(1)} | \cdots | \delta'_{\pi(n)(n)}(i)\} = 0$,

where $\delta_1|\dots|\delta_n$ is the concatenation of vectors δ_i $(1 \le i \le n)$ and $m = \sum_{1\le i\le n} | C_i |$.

A set $R = [\mu] = \{\mu' \mid \mu \simeq \mu'\}$ is called a *region* of TS. We define $R_0 = [\mu_0]$. Let R and R' be regions of TS. Then the leading relation on regions is defined as follows: $R \stackrel{a}{\to} R'$ iff $\exists \mu \in R,\ \mu' \in R'\ .\ \mu \stackrel{a}{\to} \mu'$ $(a \in Act)$; $R \stackrel{\chi}{\to} R'$ iff $\exists \mu \in R,\ \mu' \in R'\ \exists d \in \mathbf{R}^+\ .\ \mu \stackrel{d}{\to} \mu' \wedge \forall\, 0 < d' < d\ \mu \stackrel{d'}{\to} \widetilde{\mu} \in R \cup R'$.

The leading relation on regions is extended to timed words from $Dom(Act, \mathbf{R}_0^+)$ just as on the states of $ST(TS)$.

We shall call a partition of $STC(TS)$ into regions *stable* if the following holds: if $R \stackrel{a}{\to} R'$, then $\forall \mu \in R\ .\ \mu \stackrel{a}{\to} \mu'$ for some $\mu' \in R'$ $(a \in Act)$; if $R \stackrel{\chi}{\to} R'$, then $\forall \mu \in R\ \exists d \in \mathbf{R}^+\ .\ \mu \stackrel{d}{\to} \mu'$ for some $\mu' \in R'$ and $\mu \stackrel{d'}{\to} \widetilde{\mu} \in R \cup R'$ for all $0 < d' \le d$. So, we can define the notion of *region graph* of TS $RG(TS) = (V_{RG}, E_{RG}, l_{RG})$. The set of vertices V_{RG} is a stable partition of $STC(TS)$, the set of edges E_{RG} is the leading relation on regions of V_{RG}, the labelling function $l_{RG} : E_{RG} \longrightarrow Act \cup \{\chi\}$ is defined as: $l((R, R')) = z \iff R \stackrel{z}{\to} R'$.

For correctness of our formula construction we need to introduce the following notion.

Definition 6. Let $\langle w, d\rangle \in L(TS)$ and $RG(TS) = (V_{RG},\ E_{RG},\ l_{RG})$. Let $p = R_0\ \dots\ R$ be a path in $RG(TS)$. Then $\mu \in STC(TS)$ is *reachable by* $\langle w, d\rangle$ *consistent with* p iff $\mu \in R$ and either

- $p = R_0$ and $\langle w, d\rangle = \langle \epsilon, 0\rangle$,

or

- $p = p' \stackrel{z}{\to} R$ and there exists $\mu' \in STC(TS)$ reachable by $\langle w', d'\rangle$ consistent with p' and either
  - $z = a \in Act$, $\mu' \stackrel{a}{\to}\stackrel{d''}{\to} \mu$ and $\langle w, d\rangle = \langle w'a(d' - \Delta(w')), d' + d''\rangle$ for some $d'' \in \mathbf{R}_0^+$,

  or

  - $z = \chi$, $\mu' \stackrel{d''}{\to} \mu$ and $\langle w, d\rangle = \langle w', d' + d''\rangle$ for some $d'' \in \mathbf{R}^+$.

Note that $\mu_0 \stackrel{\langle w,d\rangle}{\Rightarrow} \mu \iff \forall (C, \delta) \in \mu\ (C_0, \delta_0) \stackrel{\langle w,d\rangle}{\Longrightarrow} (C, \delta)$. Moreover, for any $\langle w, d\rangle$ there exists only one $\mu \in STC(TS)$ such that $\mu_0 \stackrel{\langle w,d\rangle}{\Longrightarrow} \mu$. Consequently, R and path p from R_0 to R, such that μ is reachable by $\langle w, d\rangle$ consistent with p, are unique.

Lemma 1. *Let* $\langle w, d\rangle \in L(TS)$ *and* $\mu_0 \stackrel{\langle w,d\rangle}{\Longrightarrow} \mu$. *Then there exists only one path* p *in* $RG(TS)$ *such that* μ *is reachable by* $\langle w, d\rangle$ *consistent with* p.

Lemma 2. *Let* $R \in V_{RG}$. *Then* $\forall \mu,\ \mu' \in R\ \forall (C,\delta) \in \mu\ \exists\ (C',\ \delta') \in \mu'\ .\ C = C'\ \wedge\ S((C,\delta))\ |_{Act} = S((C',\delta'))\ |_{Act}\ \wedge S((C,\delta))\ |_{\mathbf{R}^+} = \emptyset \iff S((C',\delta'))\ |_{\mathbf{R}^+} = \emptyset$.

Let $RG(TS)$ be the region graph and X be a countable set of counters. Before we shall start to construct the formula, we need to add some additional information into common states and regions. Let all regions of $RG(TS)$ get a unique number, then with each region R_i we shall associate its own counter x_{R_i}. For simplicity, sometimes we shall denote x_{R_i} by x_i. Moreover, with each region R we shall associate the additional set of counters $RC(R)$, the region representative $\mu_R = (C_1,\ldots,C_{n_R},\delta_1,\ldots,\delta_{n_R}) \in R$ and the function $\sigma_R : RC(R) \longrightarrow 2^{n_R}$ which associates the set of configurations from μ_R with each counter of $RC(R)$. At first, we suppose $RC(R_0) = \{x_0\}$ and take μ_0 as a representative of R_0, $\sigma_{R_0}(x_0) = \{C_0\}$. For others $R \in RG(TS)$ we suppose $RC(R) = \emptyset$ and take an arbitrary $\mu \in R$ as its representative, $\sigma_R \equiv \emptyset$. Then the leading relation on regions is modified so that we add x_R into $RC(R)$, if after execution of some action we get $\mu \in R$ and some event becomes enabled in $C \in \mu$. Then the configuration C is associated with x_R. Additionally, we delete from $RC(R)$ the counters for which there are no configurations associated with them. More formally:

- $(R,RC(R)) \stackrel{a}{\to} (R',RC(R'))$ $(a \in Act)$ iff $R \stackrel{a}{\to} R'$ (suppose $\mu_R \stackrel{a}{\to} \widetilde{\mu}$ for some $\widetilde{\mu} \in R'$) and the set $RC(R')$ is modified in two steps:

  1. $RC(R') = RC(R') \cup (R \setminus OLD(R,a))$, where $OLD(R,a) = \{x_i \mid \forall j \in \sigma_R(x_i)\ .\ (C_j,\delta_j) \stackrel{a}{\not\to}\}$;
  2. $RC(R') = RC(R') \cup \{x_{R'}\}$ if $\exists e \in En(\widetilde{\mu}) \setminus En(\mu_R) \wedge \forall (C,\delta) \in \mu_R\ \forall e \in C \cup En(C)\ \delta(e) \neq 0$

  and $\sigma_{R'}$ is modified as follows:

  1. for all $x \in RC(R') \cap RC(R)$ $\sigma_{R'}(x) = \sigma_{R'}(x) \cup \{j \mid \exists i \in \sigma_R(x)\ \exists (\widetilde{C}_k,\widetilde{\delta}_k) \in \widetilde{\mu}\ .\ (C_i,\delta_i) \stackrel{a}{\to} (\widetilde{C}_k,\widetilde{\delta}_k)\ \wedge\ \exists \pi(n_{R'})\ .\ (C'_j,\delta'_j) = (\widetilde{C}_{\pi(n_{R'})(k)},\widetilde{\sigma}_{\pi(n_{R'})(k)}) \in \mu_{R'}\}$;
  2. if $x_{R'} \in RC(R')$ then $\sigma_{R'}(x_{R'}) = \{i \mid (C_i,\delta_i) \in \mu_{R'}\ .\ \exists e \in En(C_i)\ \delta_i(e) = 0\}$;

- $((R,RC(R)) \stackrel{\chi}{\to} (R',RC(R'))$ iff $R \stackrel{\chi}{\to} R'$ (suppose $\mu_R \stackrel{d}{\to} \widetilde{\mu}$ for some $d \in \mathbf{R}^+$ and $\widetilde{\mu} \in R'$) and

  – $RC(R') = RC(R') \cup (R \setminus OLD(R,\chi))$, where $OLD(R,\chi) = \{x_i \mid \forall j \in \sigma_R(x_i)(\neg\exists(\widetilde{C},\widetilde{\delta}) \in \widetilde{\mu}\ .\ (C_j,\delta_j)\ \stackrel{d}{\to} (\widetilde{C},\widetilde{\delta})\}$;

– for all $x \in RC(R') \cap RC(R)$
$\sigma_{R'}(x) = \sigma_{R'}(x) \cup \{j \mid \exists i \in \sigma_R(x)\ \exists(\widetilde{C}_k, \widetilde{\delta}_k) \in \widetilde{\mu}\ .\ (C_i, \delta_i) \xrightarrow{d} (\widetilde{C}_k, \widetilde{\delta}_k)\ \wedge\ \exists \pi(n_{R'})\ .\ (C'_j, \delta'_j) = (\widetilde{C}_{\pi(n_{R'})(k)}, \widetilde{\sigma}_{\pi(n_{R'})(k)}) \in \mu_{R'}\}$.

We also need a time assignment of our counters, so, into all common states $\mu \in R$, we include $RI_\mu = RC(R)$ and the time assignment $\Delta_\mu : RI_\mu \to \mathbf{R}_0^+$. At first, suppose $\Delta_\mu \equiv 0$. We shall omit subscription μ if it will be clear. The leading relation on common states is modified as follows:

- $(\mu, RI, \Delta) \xrightarrow{d} (\mu', RI', \Delta')$ $(d \in \mathbf{R}^+)$ iff $\mu \xrightarrow{d} \mu'$ and $\Delta' \mid_{RI} = \Delta \mid_{RI} + d$;
- $(\mu, RI, \Delta) \xrightarrow{a} (\mu', RI', \Delta')$ $(a \in Act)$ iff $\mu \xrightarrow{a} \mu'$.

It is clear that additional pieces of information have no influence on leading relations on common states and regions. In the following, we shall use a simple notation R and μ instead of $(R, RC(R))$ and $(\mu,\ RI,\ \Delta)$.

Now we can construct a formula for each region $R = [\mu_R]$. In the formula, we shall use the following notations: $R \xrightarrow{a} R_a$ and $R \xrightarrow{\chi} R_\chi$ and write its optional parts in $\langle\!\langle\rangle\!\rangle$.

$$
\begin{aligned}
F_R = \ & \forall\!\!\forall \beta(R) \Rightarrow\ \psi_R; \\
\psi_R = \ & \langle\!\langle \forall\!\!\forall \beta^{>}(R) \Rightarrow F_{nil} \rangle\!\rangle\ \wedge\ \textstyle\bigwedge_{a \notin \bigcup\{S((C,\delta)) \mid (C,\delta) \in \mu\}} [a] f\!f\ \wedge \\
& \textstyle\bigwedge_{a \in \bigcup\{S((C,\delta)) \mid (C,\delta) \in \mu\}} [a](\langle\!\langle X_a\ in \rangle\!\rangle\ \widehat{F}_{R_a})\ \wedge\ ACC(R); \\
\widehat{F}_{R_a} = \ & \begin{cases} F_R, & \text{if } \exists \mu \in R\ \exists d \in \mathbf{R}^+\ .\ \mu_R \xrightarrow{d} \mu, \\ \psi_R, & \text{otherwise.} \end{cases}
\end{aligned}
$$

Here conditions $\beta(R)$ that hold for the time assignment of states only from R are constructed in the following way:

1. $\beta(R) = t\!t$;
2. for all $x_i, x_j (x_i \neq x_j) \in RC(R)$ let $\lfloor \Delta_{\mu_R}(x_i) \rfloor = a$, $\lfloor \Delta_{\mu_R}(x_j) \rfloor = b$, then

$$\beta(R) = \beta(R) \wedge \begin{cases} x_i = a, & \text{if } \Delta_{\mu_R}(x_i) = \lfloor \Delta_{\mu_R}(x_i) \rfloor, \\ a < x_i < a+1, & \text{otherwise;} \end{cases}$$

3.

$$\beta(R) = \beta(R) \wedge \begin{cases} x_i + b = x_j + a, & \text{if } \{\Delta_{\mu_R}(x_i)\} = \{\Delta_{\mu_R}(x_j)\}, \\ x_i + b < x_j + a, & \text{if } \{\Delta_{\mu_R}(x_i)\} < \{\Delta_{\mu_R}(x_j)\}, \\ x_i + b > x_j + b, & \text{if } \{\Delta_{\mu_R}(x_j)\} < \{\Delta_{\mu_R}(x_i)\}. \end{cases}$$

The conditions $\beta^{>}(R)$ which mean that the values of counters are larger than appropriate time assignments in the states from R are constructed as follows:

$$\beta^{>}(R) = \begin{cases} \beta(R) \ \vee \ \bigvee_{x_i \in RC(R)} x_i \geq \lceil \Delta_{\mu_R}(x_i) \rceil, & \text{if all}(C,\delta) \in \mu_R \\ & \text{are terminated,} \\ \bigvee_{\{x_i \in RC(R) \mid \{\Delta_{\mu_R}(x_i)\}=0\}} x_i > \lceil \Delta_{\mu_R}(x_i) \rceil, & \text{otherwise.} \end{cases}$$

Below we give subformulas of ψ_R and conditions of including them into ψ_R.

- $X_a = \{x \mid x \in RC(R_a) \setminus RC(R)\}$ is added if it is non empty;
- $\forall\!\!\!\forall \beta^{>}(R) \Rightarrow F_{nil}$ is added into ψ_R if there is no region R_χ;
- $ACC(R) = \bigvee_{(C,\delta)\in\mu}((\bigwedge_{a\in S((C,\delta))}\langle a\rangle tt) \ \wedge \ \langle\!\langle \chi_{(C,\delta)} \ \wedge \ F_{R_\chi} \rangle\!\rangle \ \wedge \ \langle\!\langle F_{nil} \rangle\!\rangle)$;
- $F_{nil} = \bigwedge_{a\in Act}[a]ff$ is added into $ACC(R)$ for all $(C,\delta) \in \mu_R$ such that $S((C,\delta)) \mid_{Act} = \emptyset$;
- $\chi_{(C,\delta)} = \begin{cases} \exists\beta(R_\chi) \Rightarrow (\bigwedge_{a\in S((C,\delta))}\langle a\rangle tt), & \text{if } S(C,\delta) \mid_{Act} \neq \emptyset, \\ \exists\beta^{>}(R_\chi) \Rightarrow (\bigvee_{a\in Act}\langle a\rangle tt), & \text{otherwise;} \end{cases}$
- $\chi_{(C,\delta)} \ \wedge F_{R_\chi}$ is added into $ACC(R)$ for all $(C,\delta) \in \mu_R$ such that $S((C,\delta)) \mid_{\mathbf{R}^+} \neq \emptyset$.

Note that we use the symbol of implication ($\Rightarrow$) for simplicity. But it is easy to transform our formula into a correct formula from L_ν, because negation of $\beta(R)$ and $\beta^{>}(R)$ can be expressed in L_ν. Also, X_a *in* F means $(x_1 \ in \ (x_2 \ in \ (\ldots \ (x_n \ in \ F)))$ for $X_a = \{x_1, \ x_2, \ldots, \ x_n\}$. The formula ψ_R contains three obligatory groups. The first group of conjunctions contains an $[a]$-formula for any action that can not be executed in R. The second group of conjunctions contains an $[a]$-formula for any action that can be executed in R. The third group is a group of disjunctions over all states in μ_R and each disjunction part contains conjunctions of $\langle a\rangle$-formulas for each action that can be executed in some state, and an optional part which characterizes the possibility of some amount of time to pass in this state. The optional group of ψ_R is included into the formula, if there is no region R_χ.

For a timed event structure TS, *a characteristic formula* is defined as $F_{TS} = x_0 \ in \ F_{R_0}$. We have the following theorem

Theorem 1. $TS \leq_{must} TS' \iff TS' \models_D F_{TS}$, *where* D *corresponds to the previous definition of* F_R *for each* R *from* $V_{RG(TS)}$.

To prove the theorem, we need

Lemma 3. *Let* $(C'_0, \delta'_0 \ u) \models_D F_{TS}$, *where* $(C'_0, \delta'_0) = M_{TS'}$, $u \equiv 0$. *For all* $\langle w, \ d\rangle \in L(TS) \cap L(TS')$ *and* $(C'_0, \delta'_0) \stackrel{\langle w, \ d\rangle}{\Longrightarrow} (C', \delta')$ *it holds that* $(C', \delta' \ u') \models_D \psi_R$, *where* R *and* u' *are such that there exists* μ *which is reachable by* $\langle w, \ d\rangle$ *consistent with a path from* R_0 *to* R, *and* $u' \mid_{RI_\mu} = \Delta_\mu$.

Proof (Theorem 1).
($\Leftarrow$) Take an arbitrary $\langle w,\ d\rangle \in L(TS')$ and (C', δ') such that $(C'_0, \delta'_0) \stackrel{\langle w,\ d\rangle}{\Longrightarrow} (C', \delta')$. According to Definition 3, we shall show that there exists $(C, \delta) \in ST(TS)$ such that $(C_0, \delta_0) \stackrel{\langle w,\ d\rangle}{\Longrightarrow} (C, \delta)$ and $S((C, \delta)) \mid_{Act} \subseteq S((C', \delta')) \mid_{Act}$, $S((C', \delta')) \mid_{\mathbf{R}^+} = \emptyset \Rightarrow S((C, \delta)) \mid_{\mathbf{R}^+} = \emptyset$.

Assume $\langle w,\ d\rangle \notin L(TS)$. Let $\langle w,\ d\rangle = \langle a_1(d_1) \ldots a_n(d_n), \sum_{1 \leq i \leq n+1} d_i\rangle$. We can find the maximal $0 \leq k \leq n$ and $0 \leq d' \leq d_{k+1}$ for which $\langle \overline{w},\ \overline{d}\rangle = \langle a_1(d_1) \ldots a_k(d_k),\ \sum_{1 \leq i \leq k} d_i + d'\rangle \in L(TS)$. Let $\mu_0 \stackrel{\langle \overline{w},\ \overline{d}\rangle}{\Longrightarrow} \overline{\mu} \in STC(TS)$ and $(C'_0, \delta'_0) \stackrel{\langle \overline{w},\ \overline{d}\rangle}{\Longrightarrow} (\overline{C}', \overline{\delta}') \stackrel{\langle \widehat{w}, \widehat{d}\rangle}{\Longrightarrow} (C', \delta')$ for some $(\overline{C}', \overline{\delta}') \in ST(TS')$ and $\langle \widehat{w}, \widehat{d}\rangle \in Dom(Act, \mathbf{R}_0^+)$. By Lemma 1, there exists a path $\overline{p}$ in the region graph $RG(TS)$ such that $\overline{\mu}$ is reachable by $\langle \overline{w},\ \overline{d}\rangle$ consistent with $\overline{p}$. Then, by Lemma 3, $(\overline{C}', \overline{\delta}'\ \overline{u}') \models_D \psi_{\overline{R}}$ holds, where $\overline{p}$ is the path from R_0 to $\overline{R}$ and $\overline{u}' \mid_{RI_{\overline{\mu}}} = \Delta_{\overline{\mu}}$. Let us consider $\psi_{\overline{R}}$. If $d' < d_{k+1}$, then $\forall\ (C, \delta) \in \mu\ .\ S((C, \delta)) \mid_{\mathbf{R}^+} = \emptyset$, i.e., there is no region $\overline{R}_\chi$. So, by construction of the formula, $\psi_{\overline{R}}$ includes $\forall\!\!\!\forall \beta^>(\overline{R}) \Rightarrow F_{nil}$ as a conjunctive part. It is obvious that $(\overline{C}', \overline{\delta}'\ \overline{u}') \not\models_D \forall\!\!\!\forall \beta^>(\overline{R}) \Rightarrow F_{nil}$. We have got a contradiction with the assumption of Theorem 1. Similary, we can get a contradiction if $d' = d_{k+1}$. So, $\langle w,\ d\rangle \in L(TS)$.

Let $\mu_0 \stackrel{\langle w,\ d\rangle}{\Longrightarrow} \mu \in STC(TS)$. By Lemma 1 and Lemma 3, we can find R and u' such that p is a path from R_0 to R, $u' \mid_{RI_\mu} = \Delta_\mu$ and $(C', \delta'\ u') \models_D \psi_R$. By construction of the formula ψ_R and Lemma 2, there exists $(C, \delta) \in \mu$ for which $S((C, \delta)) \mid_{Act} \subseteq S((C', \delta')) \mid_{Act}\ \wedge S((C', \delta')) \mid_{\mathbf{R}^+} \Rightarrow S((C, \delta)) \mid_{\mathbf{R}^+}$.
($\Rightarrow$) Follows from construction of the formula F_{TS}. □

5. Conclusion

In this paper, we have used as a formal model a timed generalization of Winskel's prime event structures [4] which seems more appropriate for investigation of timed testing than the ones from [11, 5] because of the possibility to give notions of states and leading relation. This article is concentrated on constructing a characteristic formula. This formula allows us to decide the problem of recognizing timed *must*-equivalences by reducing it to the model-checking one. The formula obtained is only a first step towards the decision procedure for timed testing. The results may be extended onto a model with internal actions. Also, the way of construction of the characteristic formula may be applied to other timed testing equivalences.

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