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On the relationship between the electrical conductivity with other kinetic and thermodynamic parameters of sedimentary rocks*

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One of essential attributes of the method of electromagnetic monitoring of geodynamic processes is that under the action of tectonic processes, electrical properties of a geological structure change. Electrical logging data give a large body of information for the study of rocks composing a section of formations of the Earth's crust. Definition of elastic and electrical properties of sedimentary rocks is of geophysical interest. An effective solution of such problems would make it possible to essentially raise the accuracy of interpreting seismic prospecting and other geophysical data. In [1–3], it is noted that a combination of various geophysical methods increases stability and accuracy of the solution of inverse problems.

Interpretation of geophysical data, including the drilling data, allows reconstructing a detailed structure of borehole environment zones and estimating the influence of pollution of a collector with the help of a borehole mud filtrate [4]. Results of the combined geophysical and hydrodynamic interpretation can be used not only in the conventional exploration for oil, evaluation of its reservers and monitoring of oil production, but also for optimization of the technology of development of deposits, perforation of productive intervals and control of technologies of oil handling.

As is known [5], the major factors defining the electrical conductivity of sedimentary rocks are porosity and mineralization of water-saturated formations. Oil and gas deposits of a terrigenous geological structure are represented by porous collectors, the filled-in with oil and water-saturated formations. When drilling a borehole, a borehole mud filtrate penetrates into a porous space, pushing oil and a water-saturated formation deep down. This process evolves with time, and characteristics of the changed zone depend both on technological parameters of drilling and on hydrophysical features of beds (viscosity of electroconducting fluids, concentration of salts, etc.) [4]. The dependence of the electrical conductivity of rocks on their porosity was a subject of some theoretical and experimental surveys. Evidently, Archie [6] was the first to propose an empirical formula resulted from studying the core material

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$$\frac{\sigma_l}{\sigma} = d_0^{-m}, \tag{1}$$

where σ_l is the electrical conductivity of a fluid, σ is the electrical conductivity of a medium, d_0 is porosity, and m is a positive index of "consolidation".

Further, in [7], the authors extended formula (1) as follows:

$$\frac{\sigma_l}{\sigma} = ad_0^{-m}. \tag{2}$$

Here a is a stationary value depending on a medium.

In the above papers, formulas (1) and (2) are obtained under the assumption that $d_0 \geq \tilde{d}$, where $\tilde{d} = 0.1$ [6, Figures 1, 2], $\tilde{d} > 0.25$ [8, Figure 3]. Formulas (1) and (2) in the case of small porosity values are not applicable. Experimental data for certain types of rocks for small porosity values show that Archie's law is valid for the negative indices of consolidation [9].

In [4], for the simulation of the electrical conductivity around, a borehole the following empirical formula is presented:

$$\frac{1}{\sigma} = A(C_0 + C)^{-p}(S_0 + S)^{-q}(d_0 + d)^{-q},$$

where C, S are concentrations of salts and oil saturation, respectively, C, S, d are constants, the constant A, and parameters of degrees p, q being empirically selected.

In this paper, formula (1) is obtained with the negative value of the index of consolidation for the case of small values of porosity as a necessary condition of existence of a passage to the limit of the propagation velocity of seismic waves in saturated electroconducting fluid porous media in the Earth's constant magnetic field, to the propagation velocity of seismic waves in elastic media under the same conditions. Also, there is obtained a formula connecting conductivity with the streaming current coefficient C_c, as well as with dielectric permeability of a fluid ε, zeta potential ζ, porosity, permeability k, and with the ratio of the physical density of the conducting fluid ρ_l^f to the physical density of a conducting porous body ρ_s^f.

1. The relation between conductivities of a continuum and its saturated fluid for small values of porosity. In [10], a nonlinear mathematical model is constructed that combines equations of the continual filtration and the Maxwell equations, describing the motion of a conducting fluid in conducting elastic-deformed porous media. The model is based on three basic principles: the validity of conservation laws, the Galilean relativity principle, and the consistency of equations of motion for a conducting fluid with thermodynamic equilibrium conditions. It is known [11, 12] that the presence of the Earth's constant magnetic field brings in a square-law component to the propagation velocities of seismic waves in isotropic elastic media. It is theoretically shown [10] that in contrast to elastic media, in the conducting fluid-saturated porous media for small values of the Earth's constant magnetic field there occurs (along the axis x) an additional transverse seismic wave with the propagation velocity, which is proportional to the Earth's constant magnetic field

$$\pm u_A = \frac{\sigma_l}{\sigma\sqrt{d_0}} \frac{H_x}{\sqrt{4\pi\rho_l^f}},$$

where H_x is a component of the Earth's constant magnetic field along the axis x.

Hence, for small values of porosity we obtain (with linear accuracy) the formula connecting conductivities of a continuum and its saturated fluid

$$\frac{\sigma_l}{\sigma} = d_0^{0.5+n}. \tag{3}$$

Here n is an arbitrarily small positive number.

Combining formulas (1) and (3), we obtain

$$\frac{\sigma_l}{\sigma} = f(d_0), \qquad f(d_0) = \begin{cases} d_0^{0.5+n}, & d_0 \in (0, \tilde{d}), \\ d_0^{-m}, & d_0 \in [\tilde{d}, 1). \end{cases} \tag{4}$$

2. The relation between electrical conductivity of a continuum and other kinetic and thermodynamic parameters of sedimentary rocks. Laboratory experiments [13] indicate to the dependence of a streaming current coefficient on permeability. In [14, 15], the formula confirming this fact is obtained

$$C_c = \frac{\dfrac{\rho_l^f}{\rho_s^f}\varepsilon\zeta d_0}{1 - \left(1 - \dfrac{\rho_l^f}{\rho_s^f}\right) d_0} \frac{k}{\sigma_l}. \tag{5}$$

Let us exclude from formulas (4) the conductivity of the fluid used in (5) and obtain

$$\frac{\dfrac{\rho_l^f}{\rho_s^f}\varepsilon\zeta d_0}{1 - \left(1 - \dfrac{\rho_l^f}{\rho_s^f}\right) d_0} \frac{k}{\sigma C_c} = f(d_0)$$

or

$$\sigma = \frac{\dfrac{\rho_l^f}{\rho_s^f}\varepsilon\zeta}{1 - \left(1 - \dfrac{\rho_l^f}{\rho_s^f}\right) d_0} \frac{k}{C_c} \frac{d_0}{f(d_0)}.$$

Thus, we have obtained the formula connecting conductivity with kinetic and thermodynamic parameters of sedimentary rocks.

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