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On a problem for a one-dimensional non-local rock destruction system

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Abstract. In this paper, a solution to an initial boundary value problem for a one-dimensional non-local system for the destruction of rocks is constructed.

Keywords: elasticity theory, heat conduction, destruction parameter, curvature.

The technogenic deformation of massifs of rocks causes spontaneous formation of high-loaded zones around the cavities being formed. The initial destruction occurs using special facilities. As a result of an abrupt increase of stresses at the massif contour, there simultaneously develops a spontaneous disintegration process in all directions with different intensities. Deep in the massif there forms a zonal structure of stressed-deformed state (SDS) of the enclosing rocks.

An experimental investigation shows that around mines there emerges a zonal periodic structure in the form of alternating zones of destructed and relatively undestructed rocks [1–5]. This phenomenon contradicts the concepts of classical rock and massif mechanics [6], according to which a front of unlimited deformation (destruction, disintegration) moves from the mine contour deep into the massif with the formation of zones of plastic, elastoplastic, and elastic states of rocks.

This effect was probably first observed in a gold mine in South Africa (see [7, 8]). Later it was independently discovered and described in the USSR [1–5, 9, 10]. Numerous scientific papers are devoted to various aspects of this phenomenon. It is well known that in models of elastic media the metric tensor of elastic deformations characterizes the deformation energy and is a parameter of medium's state. In the process of inelastic deformation, the curvature tensor generated by the metric deformation tensor becomes nonzero. In [11] it was proposed to consider the curvature tensor as an additional parameter of the state of the medium affecting its energy (the internal energy of the medium depends on the curvature tensor invariants) and, hence, the entire deformation dynamics.

Papers [12] and [13] present the equations describing inelastic deformations taking into account the dependence of the internal energy on the curvature calculated using the metric tensor of elastic deformations. In these papers, the scalar curvature characterizing the inconsistency of deformations is called the destruction parameter. The authors have solved the problem of stationary deformations around a radial cavity and have shown that the stressed state is radially periodic. In [14], a spatially non-local model for inelastic deformations of solid bodies was obtained and investigated. The non-local character of deformations is taken into account with the help of an additional parameter of state, in additional to classical parameters, such as the tensors of stresses and deformations. This additional parameter is the curvature tensor expressed in terms of the metric tensor of deformations, and it is called the destruction parameter. In the case of small deformations, this is equivalent to the Saint–Venant incompatibility tensor. The thermodynamic properties of the model were studied, and non-stationary differential equations for a spatially non-local model consisting of the equations of the dynamic elasticity theory and the parabolic equation for the destruction parameter were formulated. The constructed model can be used to study the rock destruction problem. As an example, a one-dimensional problem of deformation of a half-plane loaded with normal stresses is considered. The statement of non-stationary problem is presented, and a qualitative agreement with the available experimental data is reached.

Let us consider a non-stationary statement of a one-dimensional problem of deformation of a half-plane $x > 0$ under the action of normal stresses. The velocity $u(t,x)$, the stresses $\sigma_{xx}(t,x)$, $\sigma_{zz}(t,x)$, and the parameter $\gamma(t,x)$ satisfy the system of one-dimensional differential equations

$$
\begin{gathered}
\rho\frac{\partial u}{\partial t} = \frac{\partial(\bar\sigma_{xx} + (\beta/\alpha)\gamma)}{\partial x},\\
\frac{\partial \bar\sigma_{xx}}{\partial t} = (\tilde\lambda + 2\mu)\frac{\partial u}{\partial x} - \psi_1\bar\sigma_{xx} - 2\psi_2\bar\sigma_{zz} - 3\frac{\beta}{\alpha}\tilde K\zeta\gamma + 4\psi_2\frac{\partial^2\gamma}{\partial x^2},\\
\frac{\partial \bar\sigma_{zz}}{\partial t} = \tilde\lambda\frac{\partial u}{\partial x} - \psi_2\bar\sigma_{xx} - \psi_3\bar\sigma_{zz} - 3\frac{\beta}{\alpha}\tilde K\zeta\gamma + 2\psi_3\frac{\partial^2\gamma}{\partial x^2},\\
\frac{\partial \gamma}{\partial t} = \beta\frac{\partial u}{\partial x} - \beta\zeta\left(\bar\sigma_{xx} + 2\bar\sigma_{zz} + 3\frac{\beta}{\alpha}\gamma\right) +\\
\frac{4}{3}\alpha\left((\zeta-\xi)\frac{\partial^2\bar\sigma_{xx}}{\partial x^2} + (\xi+2\zeta)\frac{\partial^2\bar\sigma_{zz}}{\partial x^2}\right) + 8\beta\zeta\frac{\partial^2\gamma}{\partial x^2} - \frac{8}{3}\alpha(\xi+2\zeta)\frac{\partial^4\gamma}{\partial x^4}
\end{gathered}
\tag{1}
$$

with zero initial Cauchy data and the boundary conditions [14]

$$
\sigma_{xx}|_{x=0} = -P(t), \quad \gamma|_{x=0} = 0, \quad \left.\frac{\partial\gamma}{\partial x}\right|_{x=0} = 0, \tag{2}
$$

where

$$
\begin{gathered}
\psi_1 = \tilde K\zeta + 4\mu\xi/3, \quad \psi_2 = \tilde K\zeta - 2\mu\xi/3, \quad \psi_3 = 2(\tilde K\zeta + \mu\xi/3),\\
\tilde K = \tilde\lambda + 2\,\mu/3, \quad \tilde\lambda = \lambda - \beta^2/\alpha,
\end{gathered}
$$

ρ is the density, α, β, λ, and μ are the parameters of expansion in the equation of state, and ξ, ζ are the kinetic coefficients.

Applying the Fourier transform with respect to time to both sides of relations (1) and (2) and after simple transformations, we obtain for the function $\gamma(\omega, x)$ the following sixth-order ordinary differential equation:

$$L\gamma(\omega, x) := \frac{d^6\gamma}{dx^6} + iA\frac{d^4\gamma}{dx^4} + B\frac{d^2\gamma}{dx^2} - C\gamma = 0, \tag{3}$$

$$\sigma_{xx}|_{x=0} = -P(\omega), \quad \gamma|_{x=0} = 0, \quad \frac{d\gamma}{dx}\Big|_{x=0} = 0, \tag{4}$$

where ω is the frequency,

$$A = (\psi_{10} - \psi_5\psi_9 - \psi_6\psi_8)/(\psi_5\psi_{10}), \quad B = (\psi_9 + \psi_4\psi_8 + i(\psi_5 + \psi_6\psi_7))/(\psi_5\psi_{10}),$$

$$C = (1 - \psi_4\psi_7)/(\psi_5\psi_{10}), \quad \psi_4 = \frac{3\beta\tilde{K}\zeta(2\mu\xi - i\,\omega)/\alpha}{2\tilde{K}\mu\zeta\xi/3 - \omega^2 - i\,\omega(3\tilde{K}\zeta + 2\mu\xi)},$$

$$\psi_5 = \frac{[2\mu(\psi_3 - \lambda\xi) - i\,\omega(2\mu - \lambda)]/(\rho\omega)}{2\tilde{K}\mu\zeta\xi/3 - \omega^2 - i\,\omega(3\tilde{K}\zeta + 2\mu\xi)}, \quad i = \sqrt{-1},$$

$$\psi_6 = \frac{4\omega\psi_2}{2\tilde{K}\mu\zeta\xi/3 - \omega^2 - i\,\omega(3\tilde{K}\zeta + 2\mu\xi)} + \frac{\beta}{\alpha}\psi_5,$$

$$\psi_7 = \frac{\beta\zeta(2\mu\xi - i\,\omega)}{3\frac{\beta^2}{\alpha}\zeta(2\mu\xi/3 - i\omega) - i\,\omega(\psi_3 - i\omega)},$$

$$\psi_8 = \frac{\frac{4}{3}\alpha(2\zeta\xi(\mu - \tilde{K}) - i\omega(\zeta - \xi)) + i\frac{\beta}{\rho\omega}(2\mu(\xi + 2\zeta)/3 - i\,\omega)}{3\frac{\beta^2}{\alpha}\zeta(2\mu\xi/3 - i\omega) - i\,\omega(\psi_3 - i\omega)},$$

$$\psi_9 = 2\beta\frac{i\frac{\beta}{\rho\omega}\frac{\mu}{\alpha}(\xi + 2\zeta)/3 - 2\lambda\xi\zeta}{3\frac{\beta^2}{\alpha}\zeta(2\mu\xi/3 - i\omega) - i\,\omega(\psi_3 - i\omega)},$$

$$\psi_{10} = \frac{4}{3}\frac{\alpha(\xi + 2\zeta)\left(2\omega + \frac{\beta}{\alpha}\frac{\lambda}{\rho\omega}\right)}{3\frac{\beta^2}{\alpha}\zeta(2\mu\xi/3 - i\omega) - i\,\omega(\psi_3 - i\omega)}.$$

The general solution to equation (3) has the form

$$\gamma = c_1 e^{-\nu_1 x} + c_2 e^{-\nu_2 x} + c_3 e^{-\nu_3 x}, \tag{5}$$

where ν_k are the roots with positive real parts of the corresponding characteristic polynomial of the operator L and c_k are arbitrary constants, $k = 1, 2, 3$.

Substituting (5) into (4), we obtain a non-uniform system of linear equations with respect to c_k, $k = 1, 2, 3$,

$$
\begin{aligned}
c_1 + c_2 + c_3 &= 0, \\
\nu_1 c_1 + \nu_2 c_2 + \nu_3 c_3 &= 0, \qquad (6)\\
(\psi_{11} + i\psi_{12}\nu_1^2)\nu_1^2 c_1 + (\psi_{11} + i\psi_{12}\nu_2^2)\nu_2^2 c_2 + (\psi_{11} + i\psi_{12}\nu_3^2)\nu_3^2 c_3 &= -P(\omega),
\end{aligned}
$$

where $\psi_{11} = (\psi_5\psi_9 + \psi_6\psi_8)/(\psi_5\psi_7 + i\psi_8)$, $\psi_{12} = \psi_5\psi_{10}/(\psi_5\psi_7 + i\psi_8)$.

The solution to system (6) is evident:

$$c_1 = \Delta^{-1}(\nu_2 - \nu_3)P(\omega), \quad c_2 = \Delta^{-1}(\nu_3 - \nu_1)P(\omega), \quad c_3 = \Delta^{-1}(\nu_1 - \nu_2)P(\omega).$$

Here

$$
\begin{aligned}
\Delta = {} & (\psi_{11} + i\psi_{12}\nu_1^2)\nu_1^2(\nu_3 - \nu_2) + (\psi_{11} + i\psi_{12}\nu_2^2)\nu_2^2(\nu_1 - \nu_3) + \\
& (\psi_{11} + i\psi_{12}\nu_3^2)\nu_3^2(\nu_2 - \nu_1).
\end{aligned}
$$

The component σ_{xx} of the stress tensor is determined by the formula

$$
\begin{aligned}
\sigma_{xx} = {} & (\psi_{13} + \psi_{11}\nu_1^2 + i\psi_{12}\nu_1^4)c_1 e^{-\nu_1 x} + (\psi_{13} + \psi_{11}\nu_2^2 + i\psi_{12}\nu_2^4)c_2 e^{-\nu_2 x} + \\
& (\psi_{13} + \psi_{11}\nu_3^2 + i\psi_{12}\nu_3^4)c_3 e^{-\nu_3 x},
\end{aligned}
$$

where

$$\psi_{13} = (i\psi_4\psi_8 - \psi_5)/(\psi_5\psi_7 + i\psi_8).$$

The velocity u and the component σ_{zz} of the stress tensor are determined by the formulas

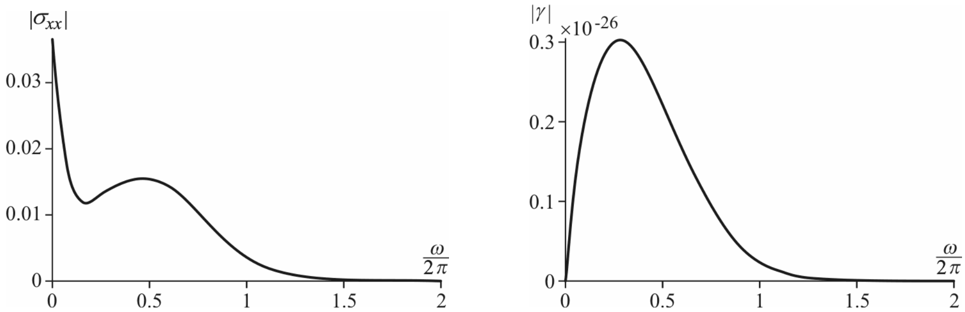
$$
\begin{aligned}
u = {} & -i\frac{\nu_1}{\rho\omega}(\psi_{13} + \psi_{11}\nu_1^2 + i\psi_{12}\nu_1^4 + \beta/\alpha)c_1 e^{-\nu_1 x} - \\
& i\frac{\nu_2}{\rho\omega}(\psi_{13} + \psi_{11}\nu_1^2 + i\psi_{12}\nu_1^4 + \beta/\alpha)c_1 e^{-\nu_2 x} - \\
& i\frac{\nu_3}{\rho\omega}(\psi_{13} + \psi_{11}\nu_1^2 + i\psi_{12}\nu_1^4 + \beta/\alpha)c_1 e^{-\nu_3 x},
\end{aligned}
$$

$$\sigma_{zz} = \frac{\psi_2}{\psi_3 - i\omega}\sigma_{xx} - 3\frac{\beta}{\alpha}\frac{\tilde{K}\zeta}{\psi_3 - i\omega}\gamma + i\frac{\lambda}{\rho\omega(\psi_3 - i\omega)}\frac{d^2\sigma_{xx}}{dx^2} + \frac{2\psi_3 + i\frac{\beta}{\alpha}\frac{\lambda}{\rho\omega}}{\psi_3 - i\omega}\frac{d^2\gamma}{dx^2}.$$

Numerical results

In this section, numerical results of the simulation of seismic wave fields for the test media models are presented. As a model, a medium consisting of a half-space loaded with normal stress is given. The medium with the following material constants was considered in [14]:

$$
\begin{gathered}
\rho = 2.2 \text{ g/cm}^3, \quad \lambda = 12.63 \text{ GPa}, \quad \mu = 4.95 \text{ GPa}, \\
\alpha = 3.14 \cdot 10^{-3} \text{ kg·m}^3/\text{s}^2, \quad \beta = -5.73 \cdot 10^1 \text{ kg·m/s}^2, \\
\xi = \zeta = 10^{-4} \text{ m·s/kg}.
\end{gathered}
$$



Amplitude spectra for $\sigma_{xx}(x,\omega)$ (left) and $\gamma(x,\omega)$ (right) for $x = 10^{-5}$ m

As a probing signal, a pulse with a bell envelope represented in the spectral domain was selected domain in the form:

$$f(\omega) = \left[\exp\left(-\left(\frac{\omega - 2\pi f}{\pi f}\right)^2\right) + \exp\left(-\left(\frac{\omega + 2\pi f}{\pi f}\right)^2\right)\right] \cdot \exp\left(\frac{-1.75\, i\, \omega}{f}\right),$$

where f is the dominant frequency equal to 1 Hz.

The results of numerical calculations of the amplitude spectra $\sigma(x,\omega)$ and the function $\gamma(x,\omega)$ at a fixed value x for $x = 10^{-5}m$ are shown in the figure. It can be seen from the figure that the spectra maximum shifts to the low-frequency domain.

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