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Thermodynamically consistent model of shale swelling around a cylindrical wellbore*

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Abstract. A modified version of the linear poroelasticity theory described by three elastic parameters is applied to shale swelling with an aqueous electrolyte. It is assumed that the shale behaves as an isotropic, ideal ionic membrane, and in this case, swelling depends only on the total stress and on the chemical water potential in pores of the rock. An analysis of flat strain around the wellbore is made.

Keywords: porous medium, saturated fluid, elastic parameters, stress tensor, partial density, Darcy law, chemical potential.

1. Introduction

The presence of pore fluids can affect the deformation process and facilitate or delay the destruction of the material [1]. The expansion of the rock under undrained deformation causes a decrease in pore pressure and an increase in the stress limit value [2]. On the other hand, the response causes an increase in pore pressure and a decrease in the stress of destruction [3].

An important mechanism for the stability of wells drilled in chemically active shale formations with water-based drilling fluids is based on the physico-chemical interactions between a rock and a drilling fluid. Namely, the pore pressure in the near-wellbore zone can be reduced due to the osmotic outflow of the pore fluid from the reactive shale, which is caused by the increased salinity of the drilling mud [4–11]. However, shales exhibit a non-ideal semi-permeable or "leaky" membrane of characteristic water-based solutions due to the range of the pore size, including wide pores, which lead to a certain permeability in salt ions. Consequently, with time, the equilibrium chemical potentials of all types in a drilling fluid and in shale formation result in a possible equalization of both pressure and chemical composition between the drilling fluid and the pore fluid near the well space [12].

The theory developed in [13, 14] for the description of coupled mechanical, hydraulic, and chemical interactions for porous bodies filled with liquid is based on the modification of Biot's poroelasticity theory [15–17]. In this paper, a thermodynamically consistent mathematical model of the linear theory of poroelasticity described by three elastic parameters applied to the shale swelling with an aqueous electrolyte is proposed.

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2. Equation of state

A mathematical model describing the interaction of a fluid flow and a change in the stress-strain state of the pore matrix was first proposed by Terzaghi [18, 19] to calculate the clay permeability coefficient. In these works, K. Terzaghi has introduced the effective stress tensor $\sigma_{ij}^{\mathrm{ef}}$, depending on the matrix deformation and fluid pressure:

$$\sigma_{ij} = \sigma_{ij}^{\mathrm{ef}} - \alpha_e p \delta_{ij}. \tag{1}$$

In formula (1), δ_{ij} are the components of the identity matrix.

Biot [17, 20] has generalized this relation to poroelastic media, where $\sigma_{ij}^{\mathrm{ef}}$ is the effective tensor of stresses (after Nur), which according to depends on the strain tensor. Sometimes ratio (1) is called the Terzagy–Biot ratio. It is actually the definition of a fractured porous medium. It has a skeleton and a fluid that saturates it. The difference from the identically zero tensor $\sigma_{ij}^{\mathrm{ef}}$ means the existence of a connected skeleton. The coefficient α_e shows how many times the pore pressure reduces the effect of the total stress on the skeleton.

The formula connecting the stress tensor with the strain tensor and the pore pressure was obtained in [21–24]:

$$\sigma_{ij} = 2G\varepsilon_{ij} + \tilde{\lambda}\varepsilon_{kk}\delta_{ij} - \beta p \delta_{ij}, \tag{2}$$

$$p = (K - \alpha\rho\rho_s)\varepsilon_{kk} - \alpha\rho\rho_l e_{kk}, \tag{3}$$

$$\varepsilon_{ij} = \frac{1}{2}\Big(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\Big), \quad e_{ij} = \frac{1}{2}\Big(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\Big),$$

where (u_1, u_2, u_3) and (U_1, U_2, U_3) are the displacement vectors of the elastic matrix and the saturating fluid with the corresponding partial densities $\rho_s = \rho_s^f(1-d_0)$ and $\rho_l = \rho_l^f d_0$, $\rho = \rho_l + \rho_s$, d_0 is the porosity, ρ_s^f and ρ_l^f are the physical densities of the elastic porous body and the liquid, respectively, $\tilde{\lambda} = \lambda - \frac{K^2}{\alpha\rho^2}$, $K = \lambda + \frac{2}{3}G$, $\beta = 1 - \frac{K}{\alpha\rho^2}$, and λ, G, $\alpha\rho^2$ are the elastic parameters of the porous medium [25].

The elastic parameters λ, G, $\alpha\rho^2$ are expressed in terms of the velocity of the propagation of the transverse wave c_s and two velocities of the longitudinal waves c_{p_1}, c_{p_2} [26, 27]:

$$\lambda = K - \frac{2}{3}G, \quad G = \rho_s c_s^2,$$

$$K = \frac{\rho\rho_s}{2\rho_l}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_l}{\rho}c_s^2 - \sqrt{(c_{p_1}^2 - c_{p_2}^2)^2 - \frac{64}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}\right),$$

$$\alpha\rho^2 = \frac{\rho^2}{2\rho_l}\left(c_{p_1}^2 + c_{p_2}^2 - \frac{8}{3}\frac{\rho_l}{\rho}c_s^2 + \frac{\rho_l - \rho_s}{\rho}\sqrt{(c_{p_1}^2 - c_{p_2}^2)^2 - \frac{64}{9}\frac{\rho_l\rho_s}{\rho^2}c_s^4}\right).$$

Note that the coefficient β, in contrast to the previously known formulas, is a function of porosity.

From (2), we express the strain tensor ε_{ij} in terms of the stress tensor σ_{ij} and the pore pressure p. Then we have

$$2G\varepsilon_{ij} = \sigma_{ij} - \frac{\tilde{\lambda}}{3\tilde{\lambda}+2G}\sigma_{kk}\delta_{ij} + \frac{2G}{3\tilde{\lambda}+2G}\beta p\delta_{ij}, \tag{4}$$

Comparing the pore pressure (3) with the pore pressure from [21–24], we obtain an expression for determining one of the four Biot parameters $R = -\alpha\rho\rho_l$. Repeating the reasoning from [28] and taking into account the thermodynamic identity for porous media, with allowance for (4) and involving the fact that $\sigma_{ij}\,d\varepsilon_{ij} + p\,dv$ is an exact differential, from (1) we obtain change in volume of pore fluid

$$v - v_0 = \frac{1}{3\tilde{\lambda}+2G}\beta\sigma_{kk} - \frac{1}{\alpha\rho\rho_l}p. \tag{5}$$

Following [28], the mass $m = \rho v$ of the pore fluid per unit volume of a material can be expressed from (1) in the linear approximation

$$\begin{aligned} m - m_0 &= (\rho - \rho_0)v + \rho_0(v - v_0) \\ &= \rho_0\frac{v_0}{K_\mathrm{f}}p + \rho_0\left[\frac{1}{3\tilde{\lambda}+2G}\beta\sigma_{kk} - \frac{1}{\alpha\rho\rho_l}p\right], \end{aligned} \tag{6}$$

where the zero index denotes the equilibrium value of the corresponding variables. In this case, the dependence of density on pressure is taken in the form [29]:

$$\frac{\rho}{\rho_0} = 1 + \frac{p - p_0}{K_\mathrm{f}},$$

where K_f is the compressibility factor of the fluid.

Further, by “dry deformation” we mean $\tilde{\Delta}m \equiv m - m_0 = 0$. In this case, from relation (6), we obtain an analogue of the Skempton formula [30] for the initial induced pore pressure and the total hydrostatic stress

$$\tilde{\Delta}p = -B\frac{\tilde{\Delta}\sigma_{kk}}{3}, \tag{7}$$

$$B = \Big(1 - \frac{K}{\alpha\rho^2}\Big)\Big(\tilde{\lambda} + \frac{2}{3}G\Big)^{-1}\Big(\frac{v_0}{K_f} - \frac{1}{\alpha\rho\rho_l}\Big)^{-1}.$$

The expression for the “dry Poisson's ratio ν_u” can be obtained by replacing for Δp from (7) in (4) and comparing the coefficients obtained with the definition of the stress tensor for an elastic body

$$2G\tilde{\Delta}\varepsilon_{ij} = \tilde{\Delta}\sigma_{ij} - \frac{\nu_u}{1+\nu_u}\tilde{\Delta}\sigma_{kk}\delta_{ij},$$

which leads to the following expression

$$\nu_u = \frac{\nu + \frac{B}{2}\beta(1-2\nu)}{1 - \frac{B}{2}\beta(1+2\nu)}.$$

In deriving this formula, we used the following formulas connecting the Poisson ratio and the elastic parameters of a porous medium

$$\nu = \frac{\tilde{\lambda}}{2(\tilde{\lambda}+G)}, \qquad \frac{\tilde{\lambda}}{3\tilde{\lambda}+2G} = \frac{\nu}{1+\nu}, \qquad \frac{2G}{3\tilde{\lambda}+2G} = \frac{1-2\nu}{1+\nu}.$$

Sometimes it is convenient to use B and ν_u instead of α, K, and $\frac{v_0}{K_f}$, since they are convenient for the physical interpretation. In fact, we can either calculate B and ν_u using other parameters, or simply take them directly from the experiment, in which the Poisson ratio and the pore pressure are measured. In terms of these coefficients, formulas (4) and (6) can be represented as

$$2G\varepsilon_{ij} = \sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{kk}\delta_{ij} + \frac{3(\nu_u-\nu)}{B(1+\nu)(1+\nu_u)}p\delta_{ij}, \tag{8}$$

$$m - m_0 = \frac{3\rho_0(\nu_u-\nu)}{2GB(1+\nu)(1+\nu_u)}\Big[\sigma_{kk} + \frac{3}{B}p\Big]. \tag{9}$$

Note that these formulas are similar in their form to [28], but there is a significant difference, namely, the Poisson ratio for a porous medium is expressed in terms of three elastic parameters of the medium. This, in turn, leads to the dependence of the Skempton coefficient B on the three parameters of a porous medium.

3. Poroelasticity theory

Further, as in [13], we consider a quasi-stationary elastic deformation, the stress tensor satisfying the equilibrium equations

$$\Delta\Big[\sigma_{kk} + \frac{6(\nu_u-\nu)}{B(1-\nu)(1+\nu_u)}p\Big] = 0, \tag{10}$$

where Δ is the Laplace operator, the repeated indices being summed from 1 to 3.

From the law of conservation of mass based on Darcy's law [24, 25], we obtain

$$\frac{\partial}{\partial t}\Big(\sigma_{kk} + \frac{3}{B}p\Big) = D\Delta\Big(\sigma_{kk} + \frac{3}{B}p\Big),$$

$$D = \frac{1}{\chi\rho(\rho_{l,0}^f)^2 d_0} \frac{2\mu(1-\nu)}{1-2\nu} \frac{B^2(1+\nu_u)^2(1-2\nu)}{9(1-\nu_u)(\nu_u-\nu)},$$

where χ is the friction coefficient.

If a porous material acts as a membrane perfect, only the chemical potential μ_w of water plays a role [13] (Sherwood 1993). Let us write down

$$\mu_w = pV_w + RT\ln a_w + \mu_w^0 + M_w gz,$$

where V_w is the partial molar volume of water, R is the gas constant, T is the temperature, and a_w is the water activity, μ_w^0 is the chemical potential in the reference state. Only differences in the chemical potential will be of interest to us, and we set $\mu_w^0 = 0$. $M_w = \rho_w V_w$ as the mass of 1 mole of water, and the gravitational potential $M_w gz$, respectively. Clearly, μ_w/V_w plays a role of a modified pressure. We will assume that V_w insignificant varies with pressure: this is inappropriate if V_w changes significantly over the pressure range of interest.

The material coefficients in the constitutive relations will be determined by the standard drained and undrained tests, and it is therefore natural to express these coefficients using the symbols G, ν, ν_u and B (or some equivalent coefficients). Thus, equations (8) and (9) become

$$2G\varepsilon_{ij} = \sigma_{ij} - \frac{\nu}{1+\nu}\sigma_{kk}\delta_{ij} + \frac{3(\nu_u-\nu)}{B(1+\nu)(1+\nu_u)} \frac{\mu_w}{V_w}\delta_{ij}, \qquad (11)$$

$$m - m_0 = \frac{3\rho_0(\nu_u-\nu)}{2GB(1+\nu)(1+\nu_u)}\Big[\sigma_{kk} + \frac{3}{B}\frac{\mu_w}{V_w}\Big]. \qquad (12)$$

The diffusion equation takes the form

$$\frac{\partial}{\partial t}\Big(\sigma_{kk} + \frac{3}{B}\frac{\mu_w}{V_w}\Big) = D\Delta\Big(\sigma_{kk} + \frac{3}{B}\frac{\mu_w}{V_w}\Big). \qquad (13)$$

We see from (12) that the parameter B now relates a change in μ_w/V_w to a change in the stress σ_{kk} in an undrained deformation, and therefore a Skempton simple extension B, which gives a change in the pore pressure p in an undrained deformation of a chemically inert system. The equation of stress equilibrium (10) becomes

$$\Delta\Big[\sigma_{kk} + \frac{6(\nu_u-\nu)\mu_w}{BV_w(1-\nu)(1+\nu_u)}\Big] = 0. \qquad (14)$$

4. Statement of the problem for a well in an infinite shale

We adopt the cylindrical coordinates, and assume that a circular wellbore of radius b is drilled through the shale along the axis z at the time instant $t = 0$.

It is assumed that the stress within the shale before drilling is uniform with the components $\sigma_{zz} = \sigma_{zz}^{\infty}$ and $\sigma_{rr} = \sigma_{\theta\theta} = \sigma_{rr}^{\infty}$, and the initial chemical potential of water within the shale is $\mu_w = \mu_w^{\infty}$ everywhere. After drilling, the boundary conditions at the wellbore will be the following:

$$\sigma_{rr} = -p_{\text{mud}}, \quad \mu_w = \mu_w^{\text{mud}}, \quad r = b,$$

where p_{mud} is the mud pressure, and

$$\mu_w^{\text{mud}} = V_w p_{\text{mud}} + RT \ln a_w^{\text{mud}} + M_w g z$$

is the chemical potential of water within the mud. The boundary conditions at infinity are $\sigma_{rr} \to \sigma_{rr}^{\infty}$, $\sigma_{\theta\theta} \to \sigma_{rr}^{\infty}$, $\sigma_{zz} \to \sigma_{zz}^{\infty}$, $\mu_w \to \mu_w^{\infty}$, as $r \to \infty$.

We take as our initial state the reference state of the rock before the well is drilled. Thus, all the stresses will be relative to the stress at infinity, and the chemical potential μ_w will be measured relative to μ_w^{∞}. The stress and chemical potential at the wellbore wall $r = b$ become the following:

$$\sigma_{rr}^b = -p_{\text{mud}} - \sigma_{rr}^{\infty}, \qquad \mu_w^b = \mu_w^{\text{mud}} - \mu_w^{\infty}.$$

Any variation of μ_w^b with depth z is assumed to be negligibly small, and deformation of the rock around the wellbore is assumed to be plane strain, with $e_{zz} = 0$. An immediate (undrained) change in the stress due to the creation of the wellbore is

$$\sigma_{rr} = -\sigma_{\theta\theta} = \frac{b^2}{r^2}\sigma_{rr}^b.$$

The subsequent deformation is controlled by the diffusion of water into the shale. Setting

$$\Phi = \sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz} + \frac{3\mu_w}{BV_w} \tag{15}$$

and taking the Fourier transform $\hat{v} = \int_R e^{i\omega t} v\, dt$ to both parts of (13) we obtain

$$D\nabla^2\hat{\Phi} = i\omega\hat{\Phi}, \tag{16}$$

The bounded solution of equation (16) is of the form

$$\hat{\Phi} = \tilde{B}(\omega) H_0^{(1)}(qr), \tag{17}$$

where $q = \sqrt{i\frac{\omega}{D}}$, $i^2 = -1$ and $H_0^{(1)}(r)$ is the Hankel function.

As is shown in [13], from the equilibrium equation (14) it follows that

$$\frac{\partial}{\partial r}\left[\sigma_{rr} + \sigma_{\theta\theta} + \sigma_{zz} + \frac{6(\nu_u - \nu)\mu_w}{BV_w(1-\nu)(1+\nu_u)}\right] = 0. \tag{18}$$

By using equations (16) and (19), we obtain

$$\frac{3(1-\nu_u)(1+\nu)\bar{\mu}_w}{BV_w(1-\nu)(1+\nu_u)} = \tilde{B}(\omega)H_0^{(1)}(qr),$$

and the boundary condition $\mu_w = \mu_w^b$ at $r = b$ implies

$$\tilde{B}(\omega) = \frac{3(1-\nu_u)(1+\nu)\hat{\mu}_w^b}{BV_w(1-\nu)(1+\nu_u)H_0^{(1)}(qb)}.$$

Equations (11), (15), (18) and the relations $u_r = u(r,t)$, $e_{rr} = \dfrac{\partial u}{\partial r}$, $e_{\theta\theta} = \dfrac{u}{r}$ result in

$$\hat{u}r = \frac{\eta\hat{\mu}_w^b r H_1^{(1)}(qr)}{GV_w q H_0^{(1)}(qb)} + \frac{C_2(\omega)}{2Gr},$$

where

$$\eta = \frac{3(\nu_u - \nu)}{2B(1+\nu_u)(1-\nu)}$$

and $C_2(s)$ is a constant of integration. The radial stress is determined by the expression

$$\begin{aligned}\hat{\sigma}_{rr} &= 2G\hat{e}_{rr} + \frac{\nu}{1+\nu}\hat{\sigma}_{kk} - \frac{3(\nu_u-\nu)\hat{\mu}_w}{BV_w(1+\nu)(1+\nu_u)} \\ &= \frac{b^2}{r^2}\hat{\sigma}_{rr}^b + \frac{2\eta\hat{\mu}_w^b}{V_w q}\left[\frac{bH_1^{(1)}(qb)}{r^2H_0^{(1)}(qb)} - \frac{H_1^{(1)}(qr)}{rqH_0^{(1)}(qb)}\right],\end{aligned}$$

where, to satisfy the stress boundary condition at the wellbore, we have set

$$C_2(\omega) = -\frac{2\eta\hat{\mu}_w^b bH_1^{(1)}(qb)}{V_w q H_0^{(1)}(qb)} - b^2\hat{\sigma}_{rr}^b.$$

The tangential stress $\sigma_{\theta\theta}$ is given by the formula

$$\begin{aligned}\hat{\sigma}_{\theta\theta} &= 2G\hat{e}_{\theta\theta} + \frac{\nu}{1+\nu}\hat{\sigma}_{kk} - \frac{3(\nu_u-\nu)\hat{\mu}_w}{BV_w(1+\nu)(1+\nu_u)} \\ &= -\frac{b^2}{r^2}\hat{\sigma}_{rr}^b + \frac{2\eta\hat{\mu}_w^b}{V_w}\left[\frac{H_1^{(1)}(qr)}{rqH_0^{(1)}(qb)} - \frac{bH_1^{(1)}(qb)}{r^2qH_0^{(1)}(qb)} - \frac{H_0^{(1)}(qr)}{H_0^{(1)}(qb)}\right]\end{aligned}$$

hence,

$$\hat{\sigma}_{rr} - \hat{\sigma}_{\theta\theta} = \frac{2b^2}{r^2}\hat{\sigma}_{rr}^b + \frac{2\eta\hat{\mu}_w^b}{V_w}\left[\frac{2bH_1^{(1)}(qb)}{r^2qH_0^{(1)}(qb)} - \frac{2H_1^{(1)}(qr)}{rqH_0^{(1)}(qb)} + \frac{H_0^{(1)}(qr)}{H_0^{(1)}(qb)}\right].$$

the axial stress is

$$\sigma_{zz} = \frac{\nu}{1+\nu}\Phi - \frac{3\nu_u\mu_w}{BV_w(1+\nu_u)}, \qquad \hat{\sigma}_{zz} = \frac{2\eta\hat{\mu}_w^b H_0^{(1)}(qr)}{V_w H_0^{(1)}(qb)}.$$

5. Numerical results

The displacement and stress due to the constant σ_{rr}^b do not change with time:

$$u = -\frac{b^2}{2Gr}\sigma_{rr}^b, \qquad \sigma_{rr} = -\sigma_{\theta\theta} = \frac{b^2}{r^2}\sigma_{rr}^b$$

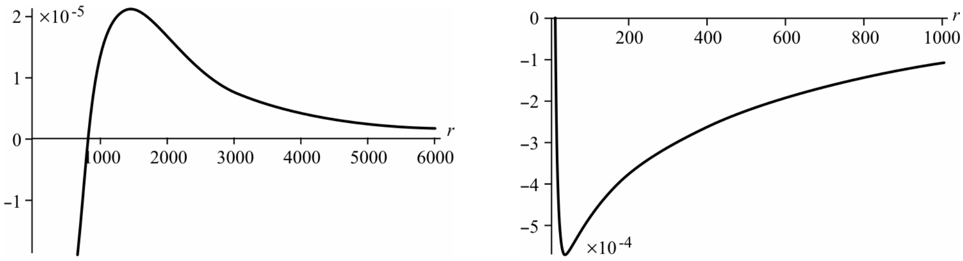
and the deviatoric stress is

$$\sigma_{rr} - \sigma_{\theta\theta} = \frac{2b^2}{r^2}\sigma_{rr}^b.$$

This is the load of mode 1 in the notation from [16].

The fluid invasion occurs if $\mu_w^b > 0$ and draining occurs if $\mu_w^b < 0$. Setting $\sigma_{rr}^b = 0$, we obtain the load of mode 2 [16].

Figures 1–3 show the numerical results of modeling the dimensionless components of the stress tensor $\tilde{\sigma}_{ij} = \sigma_{ij}/(\eta\mu_w^b/V_w)$ due to the load of mode 2 ($\sigma_{rr}^b = 0$) for test models of media. As a model, a uniform porous medium was set. Physical characteristics were set as follows: $\rho_s^f = 1.5$ g/cm^3, $\rho_s^f = 0.9$ g/cm^3, $G = 60 \cdot 10^9$ din/cm^2, $\chi = 10^5$ cm^3/(g·s), $d_0 = 0.2$, $\nu = 0.2$, $\nu_u = 0.33$, $B = 0.62$ [28], $b = 10$ cm.



a b

Figure 1. The dimensionless stress $\tilde{\sigma}_{rr}$ as a function of radius at the frequency $f = 2$Hz: a) the real part, b) the imaginary part

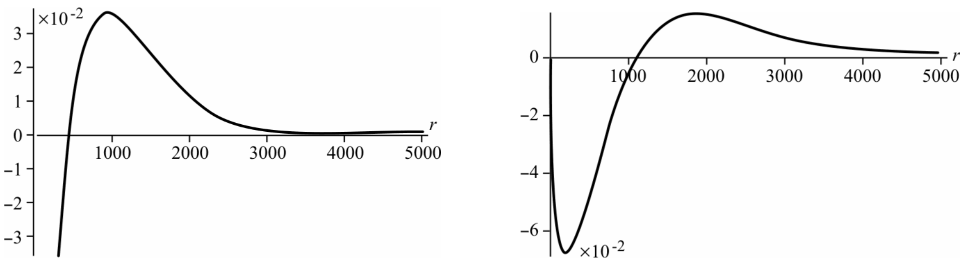


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Figure 2. The dimensionless stress $\tilde{\sigma}_{\theta\theta}$ as a function of radius at the frequency $f = 2$Hz: a) the real part, b) the imaginary part

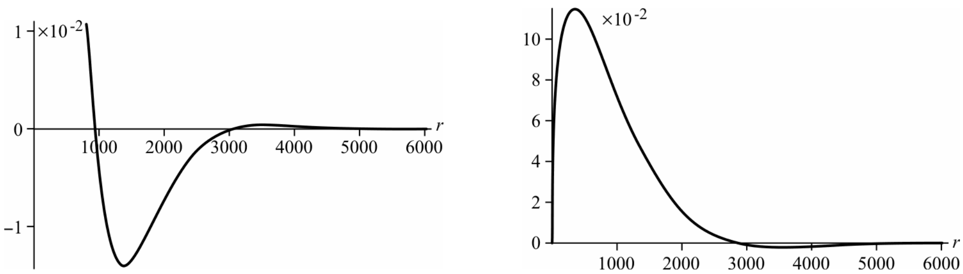

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Figure 3. The dimensionless stress $\tilde{\sigma}_{zz}$ as a function of radius at the frequency $f = 2$Hz: a) the real part, b) the imaginary part

Conclusion

A modified version of the linear poroelasticity theory described by three elastic parameters for studying the shale swelling with an aqueous electrolyte is proposed. The analysis of a flat strain around the wellbore has been made.

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