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Kolpanskoye Lake near St. Petersburg. Origin hypothesis

K.M. Klyaus

Abstract. A hypothesis is formulated about the formation of Kolpanskoye Lake, located in the vicinity of the city of Gatchina (near St. Petersburg, Russia) as a result of a meteorite fall. An approximate calculation of meteorite characteristics was performed using the energy model of the meteorite/ground dynamic system and nonlinear programming methods. The obtained preliminary data can be used in further research.

Keywords: meteorite, impact crater, energy model, nonlinear programming.

Introduction

It is known that the main morphostructural features of a simple meteorite crater are the following: a bowl-shaped form, the presence of outer rings of hills, and small diameters (up to 2–3 km in sedimentary rocks) [1–3]. Similar signs seen in the surface relief and visual dimensions of any natural object give the right to start studying it from the point of view of its possible impact origin.

The city of Gatchina, in our days the capital of the Leningrad region, is located 45 kilometers south of St. Petersburg (Figure 1).

Gatchina became an Imperial Residence under Paul I in 1796, and since that time has played and still plays a special role in the history of Russia. At the end of the 18th century, the city center was the Imperial Palace and the Park Ensemble, which included three park areas: the Palace Park, the Menagerie and the Priory Park, as well as a system of beautiful lakes, rivers and canals. The water system created by Vincenzo Brenna, Nikolay A. Lvov and other outstanding architects is a harmony of natural and artificial reservoirs. The most important element of the water system of the Park Ensemble is Kolpanskoye Lake (Figure 2).

The almost circular shape of the lake and its relatively small size allow us to suggest that the lake was formed as a result of a meteorite impact. So far, this assumption is based only on the shape of a lake typical for impact craters [1–4]. Whether the poorly visible hills located on the western and northern shores of the lake are fragments of a preserved crater ring is not known up today.

A computational forecast of the meteorite characteristics for the conditions of impact, which are usually accepted as a initial rough estimate, has

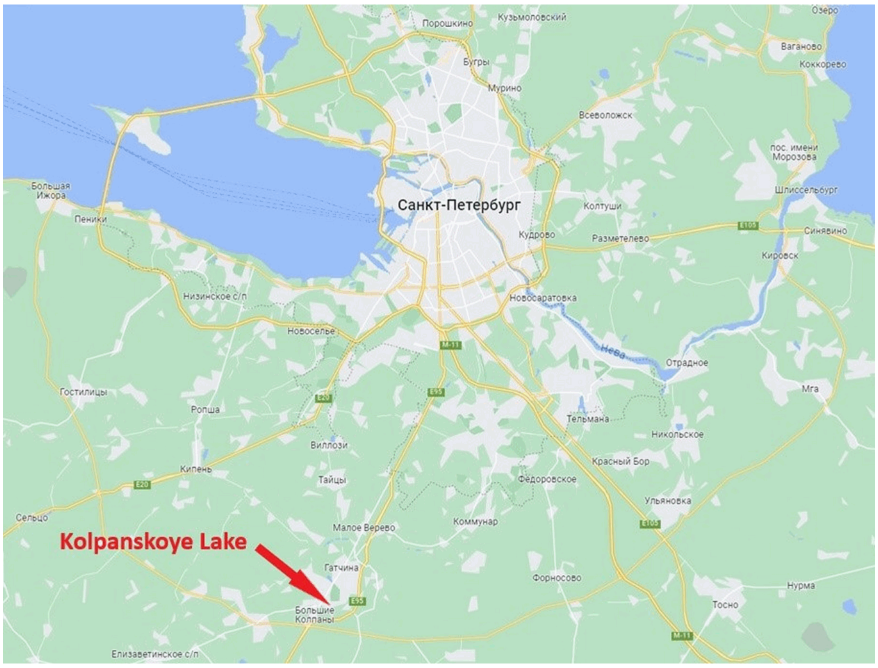

Figure 1. Kolpanskoye Lake near St. Petersburg
(URL: http://www.google.com/maps)

Figure 2. Kolpanskoye Lake (author's photo, July 2023)

been made. To calculate the diameter and velocity of the meteorite, we used a technique based on the representation of the diameter and depth of the transitional crater as a function of the energy and momentum of a cosmic body. The problem is reduced to a nonlinear programming problem, the discrepancy between the calculated and actual values of the diameter of the transitional crater is taken as the objective function.

1. Kolpanskoye Lake. A brief information

Kolpanskoye Lake is the main source of water for the flowing lakes of the Gatchina Park Ensemble and is a natural elliptical reservoir with a small ratio of axes 1.2 : 1.0 (Figure 3). The characteristic size—the width (diameter) of the lake according to the average water level is approximately equal to 600 m, the depth is from 1 to 3 m. The area of the lake is 0.32–0.37 square kilometers.

The thickness of the layer of bottom silty sediments, according to various estimates, is about 8 m. The shores are low and swampy, overgrown with reeds. There are almost no shrubs and trees on the northern shore and dense vegetation prevails on the southern shore. The lake is fed by ground water and surface runoff from the catchment area. In summer, a significant part of the lake is covered with aquatic vegetation. From the lake, in addition to the Kolpanka River, the artificial Kolpanskiy Canal flows out. Both waterways connect the lake with a lake system of the Palace and Priory parks.

Figure 3. Kolpanskoye Lake.
The satellite photo
(URL: http://www.google.com/maps)

2. Calculation technique

As the initial model of the dynamic system meteorite/surface of the Earth's crust, an approximate energy model was adopted, which relates the mass (linear dimensions and density) of the meteorite with the diameter and depth of the transitional crater and ground properties. The model proposed by N.I. Shishkin, involves solving a direct problem: determining the sizes of the transitional crater as a function of the known characteristics of the meteorite [5]:

$$D_t = \left(\frac{E}{\tau_s}\right)^{0.255} \left(\frac{I}{\rho_t c_t}\right)^{0.078} + 1.20\sqrt{\frac{\rho_p}{\rho_t}} d_p, \tag{1}$$

$$h_t = 0.14\left(\frac{E}{\tau_s}\right)^{0.255} \left(\frac{I}{\rho_t c_t}\right)^{0.078} + 0.60\sqrt{\frac{\rho_p}{\rho_t}} d_p, \tag{2}$$

where

- D_t, h_t are diameter and depth of the transitional crater, m;
- d_p is meteorite diameter (largest linear size), m;
- $E = m_p v^2/2$ is impact kinetic energy, J [kg $\cdot$ (m/s)^2];
- $I = m_p v$ is momentum, kg $\cdot$ (m/s);
- $m_p = \pi \rho_p d_p^3/6$ is meteorite mass, kg;
- $v = 10^3 v_p \sin\theta$ is vertical component of meteorite velocity at the point of impact, m/s;
- v_p is absolute velocity of the meteorite at the point of impact, km/s;
- θ is angle of incidence relative to the horizon plane;
- $\tau_s = \tau_t + kgd_p\sqrt{\rho_p\rho_t}$ is shear stress in the ground at penetration depth $h_p = d_p\sqrt{\rho_p/\rho_t}$, Pa;
- τ_t is ground cohesion stress, Pa;
- k is ground cohesion coefficient;
- g is gravitational acceleration at latitude φ, m/s^2;
- ρ_p, ρ_t are meteorite and ground densities, kg/m^3;
- c_t is wave propagation speed in the ground, m/s.

Determining the meteorite parameters as a function of the size of the transitional crater is possible on the basis of solving the inverse problem. For this purpose, in this paper, it is proposed to use the methods of nonlinear programming [6].

Two parameters, i.e., the meteorite diameter d_p and the absolute velocity at the impact point v_p should be taken as independent variables. This is sufficient for an approximate estimate of these parameters, and since the objective function of two variables is a surface, the visibility of the solution is provided.

Thus, in terms of nonlinear programming, the problem is formulated as follows: to minimize the discrepancy between the calculated and actual values of the diameter of the transitional crater $F(x)$ on the set of independent variables $R(x)$, where

$$x = (x_1, x_2)^T, \quad x_1 \equiv d_p, \quad x_2 \equiv v_p,$$

subject to the constraints:

$$x_1^{\min} \le x_1 \le x_1^{\max}, \quad x_2^{\min} \le x_2 \le x_2^{\max}.$$

Additionally, the v_p parameter is treated as a random variable obeying a lognormal probability distribution (the author does not know such statistics; in the first approximation, an analogy with probability distribution the masses of meteorites and bolides is accepted [7]). This assumption requires clarification or even a special study.

Taking equation (1) as a basis and performing some transformations, the objective function can be represented as follows:

$$F(x) = \frac{p(x_2)}{p(v_p)} - 1 + \left[A_4 + A_3^{0.0783}\Big(\frac{A_1}{\tau_t + A_2 x_1}\Big)^{0.2551} x_2^{0.5885} - \frac{D_t}{x_1}\right]^2, \tag{3}$$

under the following constraints on the independent variables

$$18 \le x_1 \le 60, \quad 12 \le x_2 \le 72.$$

Expression (3) uses the following notations:

$$A_1 = 2.618 \cdot 10^5 \cdot \sin^2\theta \cdot \rho_p, \quad A_2 = kg\sqrt{\rho_p\rho_t},$$
$$A_3 = 5.236 \cdot 10^2 \cdot \sin\theta \cdot \sqrt{\frac{\rho_p}{\rho_t}} \cdot \frac{1}{c_t}, \quad A_4 = 1.20\sqrt{\frac{\rho_p}{\rho_t}},$$
$$g = 9.780318 \cdot (1 + 0.005302 \cdot \sin^2\varphi - 0.000006 \cdot \sin^2 2\varphi) - 0.000003086 \cdot h.$$

Assuming the latitude and height above sea level equals to $\varphi = 59°32'11''$ and $h = 92$ m, we get $g = 9.81854733819659$ m/s^2.

The probability density function (PDF) $p(y)$ of the lognormal distribution of meteorite velocity is defined as follows [8]:

$$p(y) = \frac{1}{y\sigma\sqrt{2\pi}} \cdot \exp\Big[-\frac{(\ln y - \mu)^2}{2\sigma^2}\Big].$$

When deriving the formula for the PDF $p(y)$ in relation to the case under consideration, the following parameter values were obtained:

$$\mu = 0.083709268126, \quad \sigma = 0.95,$$

corresponding to the most probable velocity of meteorite taken equal to $v_p = 20$ km/s (in this case $y = 0.05v_p - 0.55$).

Finally, we obtain the following expression for the PDF $p(y)$ with $y = 0.05x_2 - 0.55$:

$$p(x_2) = \frac{1}{2.381296840589 \cdot (0.05x_2 - 0.55)} \times \exp\Big[-\frac{(\ln(0.05x_2 - 0.55) - 0.083709368126)^2}{1.805}\Big]. \tag{4}$$

As follows from expressions (3), (4), the solution of the nonlinear programming problem requires the use of numerical optimization methods.

Studies have shown that the objective function in the tolerance range of independent variables refers to smooth functions and has no discontinuity points. In this case, the solution of the problem is not difficult and can be performed using any iterative methods. In this work, the solution was obtained by the Generalized Reduced Gradient Method using the Microsoft Excel application.

The achieved accuracy of the solution ($|\delta F(x)| < 1 \cdot 10^{-3}$) is quite sufficient both for the initial assessment and for subsequent refinements, for which the order of magnitude is important. Talking about a high accuracy is irrelevant.

3. Model estimations of meteorite and transitional impact crater parameters

To perform an approximate computational assessment of the characteristics of the meteorite, the initial data presented in Table 1 were taken.

The objective function (3) for the considered cases of iron and stone meteorite is presented in Figures 4 and 5. The results of calculating the

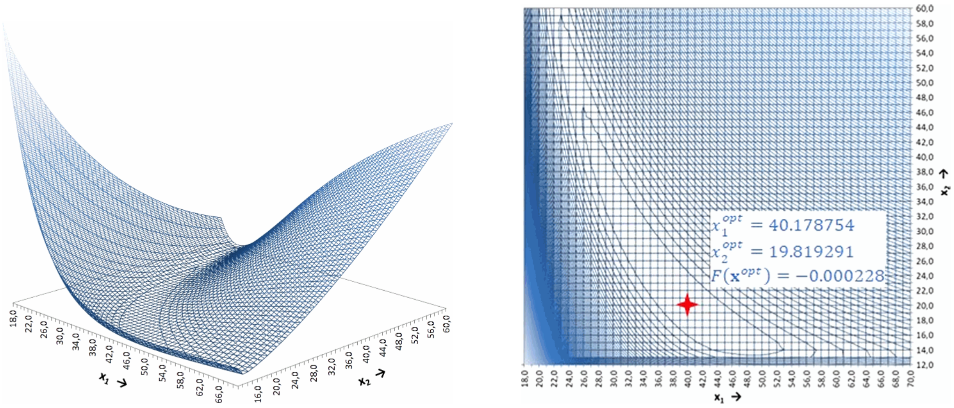

Figure 4. Iron meteorite: axonometric view (left), isolines and solution (right)

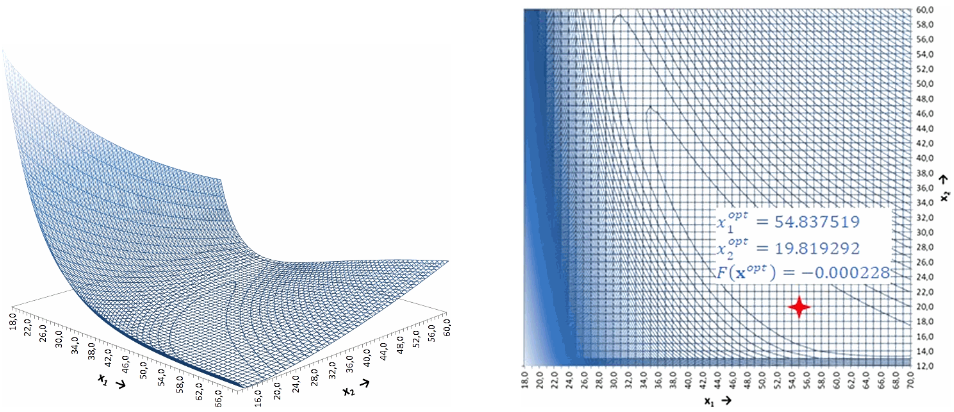

Figure 5. Stone meteorite: axonometric view (left), isolines and solution (right)

Table 1. Initial data

Parameter denomination	Symbol	Units	Value (Iron Stone)	Reference
Lake/crater diameter	D	m	600	
Transitional crater diameter	$D_t = 0.84D$	m	500	[3]
Ground cohesion coefficient	k	–	1.0	[5]
Wave propagation speed in the ground	c_t	m/s	4780	[5]
Ground density	ρ_t	kg/m^3	2840	[5]
Ground cohesion stress	τ_t	MPa	50	[5]
Meteorite density	ρ_p	kg/m^3	7600 2900	[5]
Angle of incidence	θ	deg.	45	

Table 2. Calculated meteorite parameters

Parameter denomination	Symbol	Units	Value (Iron Stone)	Reference
Estimated absolute velocity of the meteorite at the point of impact (calculated value)	v_p	km/s	20 20	Section 2
Vertical component of ground entry velocity (calculated value)	$v = v_p \sin\theta$	km/s	14	
Estimated diameter	d_p	m	40 55	
Estimated mass	m	kt	255 250	
Kinetic impact energy	$E = mv^2/2$	MJ	$\approx 2.5 \cdot 10^{10}$	
Momentum	$I = mv$	kg·m/s	$\approx 3.6 \cdot 10^{12}$	

Table 3. Calculated estimate of the crater depth

Parameter denomination	Symbol	Units	Value (Iron Stone)		Reference
Lake/crater diameter	D	m	600		Table 1
Transitional crater diameter	D_t	m	500		Table 1
Transitional crater depth	h_t	m	45	39	(2)
Depth of the transitional crater, taking into account the layer of breccia	$h \approx 0.67 h_t$	m	30	25	[3]
Depth to diameter ratio	h/D	–	0.050	0.042	

meteorite parameters are given in Table 2. An estimate of the crater depth is provided in Table 3.

Thus, for the velocity of a meteorite near the ground taken equal to 20 km/s, its mass can be about 250 thousand tons at an angle of incidence $\theta = 45°$. The forecast of kinetic energy gives the order of magnitude of $2.5 \cdot 10^{10}$ MJ.

Calculations were also performed for other initial data, in particular, for the angle of incidence $\theta = 30°$. In this case, the mass (either for iron and stone meteorite) at the same velocity would be 420–425 thousand tons. The diameter of the transitional crater would retain its value, but the depth would decrease by 13 %.

Conclusion

In this paper, an assumption is made about the origin of Kolpanskoye Lake within the meteorite impact crater. Probably, the crater (if this assumption is correct) appeared long before the last Ice Age. Over a long period of time, its original form has undergone significant changes. Not only prolonged soil erosion, but also the last glaciation could have a significant impact on this process.

An approximate calculation of the meteorite size and velocity is offered. The calculations are based on an approximate energy model that connects the mass (linear dimensions and density) and velocity of the meteorite with the diameter and depth of the transitional crater and ground properties. The desired parameters (meteorite diameter and velocity) are treated as independent variables and are found using one of the numerical optimization methods. The proposed calculation method can be used in similar studies.

It is obvious that the calculated estimate is only methodological sample and insufficient for confirming or refuting the assumptions. In this regard, it seems appropriate for interested scientific institutions, both St. Petersburg and the Siberian Branch of the RAS, to organize a series of field studies to identify various signs of impact metamorphism, including gravity anomalies.

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