

# Modeling of a pollutant spreading from high sources in atmosphere\*

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## 1. Introduction

The need to estimate an influence of high sources to the air quality near the land surface by mathematical methods leads to the construction of models describing the processes of a pollutants spreading in atmosphere. One peculiarity of this problem is the following. The source of a pollutant arranges on the height, where the horizontal component of wind velocity vector can take a large value. Therefore, the pollution spreads to the considerable distance before it reaches the lower layer of the atmosphere. Thereby, firstly, we have to consider a large domain for modeling this process. But we can not solve this problem using sufficiently small grid steps owing to computer possibilities. Secondly, the great interest is to study the features formed in concentration field as a result of influence of underlying surface. In this case, we often want to see a comprehensive scene in some small domain. An ordinary interpolation of modeling results obtained by using a large-step grid in the large domain apparently does not give us a probable enough scene.

In this paper, one of the approaches to solve this problem is considered.

## 2. Numerical model

The numerical models of hydrodynamics and transport of pollutants in the atmosphere of limited territory were discussed in papers [1, 2].

Let us consider two domains. We need to estimate an influence of high source to the air quality near the land surface in the first domain. The second domain contains the first one. The horizontal size of this domain is determined by the corresponding size of the first domain, the typical value of the wind velocity and the rate of the gravitational settling of a pollution.

Let us consider the suggested method. In each time step, we solve the hydrodynamics and transport of pollutants in the atmosphere problem in

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\*Supported by the Russian Foundation for Basic Research under Grants 98-05-65318, 99-07-90422, 00-15-98543 and by the Program of Advanced Informative Technology of the Russian Ministry and Technology under Grant 0201.06.269/349.

the second domain. This time we use a large horizontal grid steps. We interpolate obtained solutions on the boundary of the first domain and use them as a boundary conditions in the time of solving the same problem on the small-steps grid in this domain. In this case, the model can be considered as an interpolant.

By way of illustration, let us consider some methodics of numerical experiments. We use the model that was represented in paper [3]. The system of equations describes the atmospheric hydrodynamics over inhomogeneous surface in hydrostatic approximation. For the convenient relief description the vertical coordinate is used which follows the relief

$$\sigma = \frac{p - p_T}{\pi_s}, \quad \pi_s \equiv p_s - p_T, \quad (1)$$

here  $p$  is the pressure,  $p_T$  and  $p_s$  are pressures at the upper air boundary and at the land surface. The coordinates  $x$  and  $y$  are directed to the east and the north accordingly.

The basic equations of the model are the following [2, 3]:

The equations of motion

$$\frac{\partial \pi_s u}{\partial t} + \tilde{\mathcal{L}}(\pi_s u) - l \pi_s v = -\pi_s \left[ \frac{\partial H}{\partial x} + \frac{\sigma RT}{\Phi} \frac{\partial \pi_s}{\partial x} \right], \quad (2)$$

$$\frac{\partial \pi_s v}{\partial t} + \tilde{\mathcal{L}}(\pi_s v) + l \pi_s u = -\pi_s \left[ \frac{\partial H}{\partial y} + \frac{\sigma RT}{\Phi} \frac{\partial \pi_s}{\partial y} \right], \quad (3)$$

where  $\Phi \equiv \sigma \pi_s + p_T$ ;

The continuity equation

$$\frac{\partial \pi_s}{\partial t} + \mathcal{L}(\pi_s) = 0. \quad (4)$$

Here

$$\mathcal{L}(\pi_s \varphi) = \frac{\partial \pi_s \varphi u}{\partial x} + \frac{\partial \pi_s \varphi v}{\partial y} + \frac{\partial \pi_s \dot{\sigma} \varphi}{\partial \sigma} \quad (5)$$

is the transport operator in  $\sigma$ -system of coordinates in the divergent form;

The equation

$$\tilde{\mathcal{L}}(\pi_s \varphi) = \mathcal{L}(\pi_s \varphi) + F_\varphi^H + F_\varphi^B, \quad (6)$$

where  $F_\varphi^H$  and  $F_\varphi^B$  are the operators of the turbulent exchange of a substance  $\varphi$  in the horizontal and vertical directions,  $\vec{u} = (u, v, \dot{\sigma})$  is the wind velocity vector,  $u$ ,  $v$ , and  $\dot{\sigma}$  are the components of the velocity vector in the directions of  $x$ ,  $y$ , and  $\sigma$ , respectively,  $\dot{\sigma} = d\sigma/dt$ .

The equation for the pressure tendency  $\pi_s \equiv p_s - p_T$  is the following:

$$\frac{\partial \pi_s}{\partial t} + \int_0^1 \left[ \frac{\partial \pi_s u}{\partial x} + \frac{\partial \pi_s v}{\partial y} \right] d\sigma = 0. \quad (7)$$

It was derived by vertical integration of the continuity equation (4) under conditions  $\dot{\sigma} = 0$  at  $\sigma = 0 (p = p_T)$  and  $\sigma = 1 (p = p_s)$ .

The equation for the vertical component in  $\sigma$ -coordinates:

$$\dot{\sigma} = -\frac{1}{\pi_s} \int_0^\sigma \left[ \frac{\partial \pi_s}{\partial t} + \frac{\partial \pi_s u}{\partial x} + \frac{\partial \pi_s v}{\partial y} \right] d\sigma. \quad (8)$$

The expression  $\partial \pi_s / \partial t$  is excluded with the help of (7).

The equation for the heat income:

$$\frac{\partial \pi_s T}{\partial t} + \tilde{\mathcal{L}}(\pi_s T) - \frac{RT\tau}{c_p(\sigma + p_T/\pi_s)} = \frac{\pi_s Q}{c_p}, \quad (9)$$

$$\tau \equiv \frac{dp}{dt}, \quad \tau = \pi_s \dot{\sigma} + \sigma \frac{d\pi_s}{dt}, \quad \frac{d\pi_s}{dt} = \frac{\partial \pi_s}{\partial t} + u \frac{\partial \pi_s}{\partial x} + v \frac{\partial \pi_s}{\partial y}, \quad (10)$$

where  $T$  is the temperature,  $c_p$  is a specific heat at a constant pressure,  $Q$  is a heat source term.

The hydrostatic equation has the form

$$\frac{\partial H}{\partial \sigma} = -\frac{\pi_s R}{\Phi} T. \quad (11)$$

The equation of the transport of pollution is the following:

$$\frac{\partial \varphi}{\partial t} + \tilde{\mathcal{L}}(\varphi) = f, \quad (12)$$

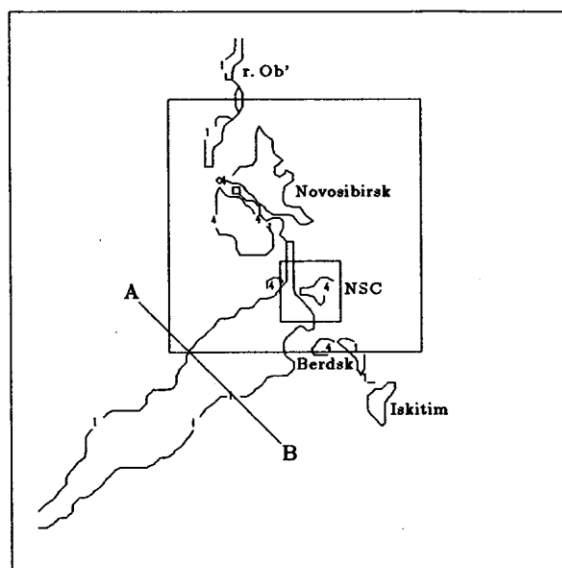
where  $f$  is the function describing the sources of pollution,  $\varphi$  is the concentration of the pollutant. In general, pollutants are multi-component mixture. The number of components is prescribed as an initial parameter of the model.

The rates of the gravitational settling are taken into account by adding the corresponding values to the vertical component of the velocity vector.

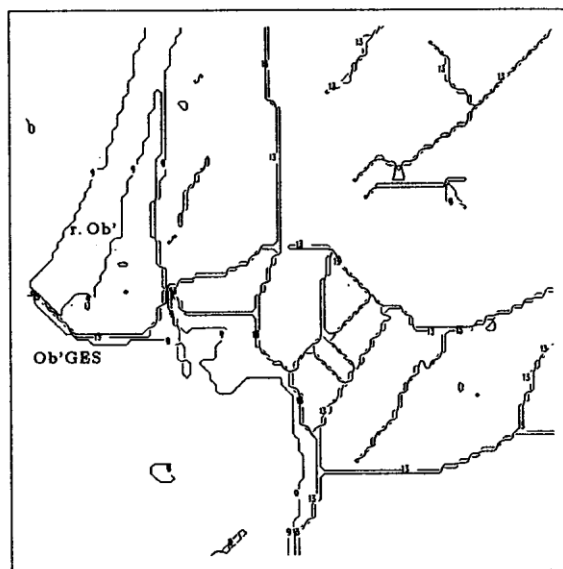
### 3. Numerical experiment

We take the Novosibirsk Scientific Center (NSC) and the Novosibirsk Industrial Region (NIR) as the first and second domains accordingly. We also consider the so-called Novosibirsk Industrial Area (NIA) as the additional domain (Figures 1 and 2).

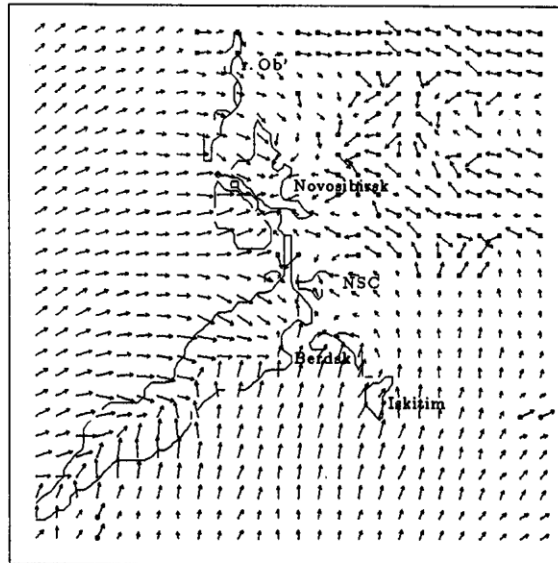
In each time step, we solve the hydrodynamics and transport of pollutants in the atmosphere problem in the NIR. Then we interpolate obtained solutions on the boundary of the NIA and use them as boundary conditions in the time of solving the same problem. The results are interpolated in the



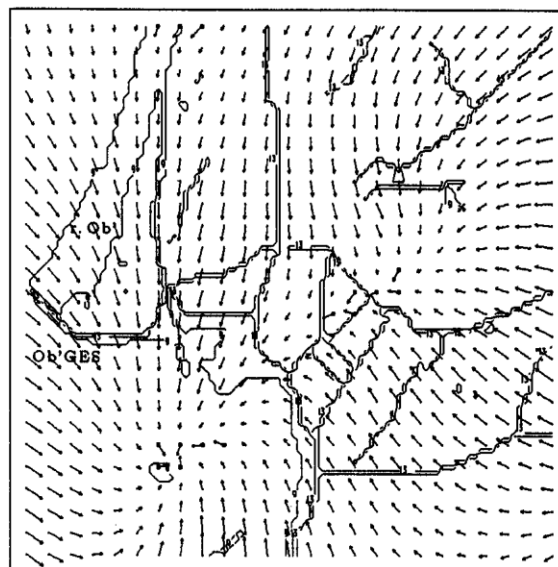
**Figure 1.** Novosibirsk Industrial Region ( $100 \times 100$  km); Novosibirsk Industrial Area ( $50 \times 50$  km); Novosibirsk Scientific Center ( $12.5 \times 12.5$  km); 4 marks the boundary of town territory, 1 marks the boundary of water basin



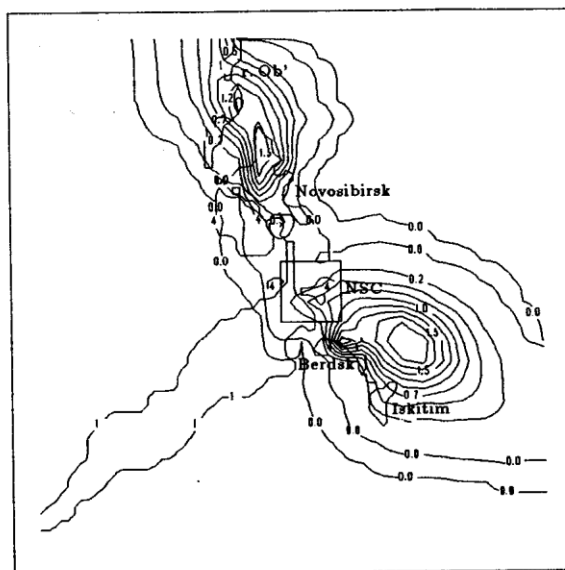
**Figure 2.** Novosibirsk Scientific Center ( $12.5 \times 12.5$  km); 13 marks roads, 9 marks the boundary of water basin



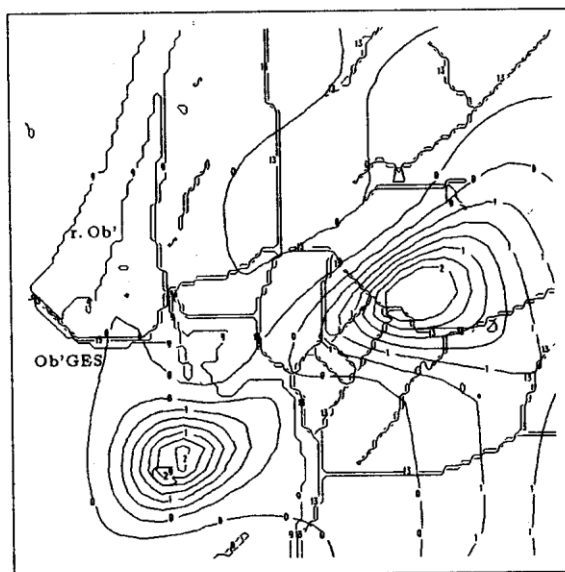
**Figure 3.** Novosibirsk Industrial Region. Horizontal structure of atmospheric circulation (height 50 m). Autumn, 17.00 of local time, south-western background flow



**Figure 4.** Novosibirsk Scientific Center. Horizontal structure of atmospheric circulation (height 50 m). Autumn, 17.00 of local time, south-western background flow



**Figure 5.** Novosibirsk Industrial Region. Horizontal section at 50 m of concentrations field. Autumn, 17.00 of local time, south-western background flow



**Figure 6.** Novosibirsk Scientific Center. Horizontal section at 50 m of concentrations field. Autumn, 17.00 of local time, south-western background flow

NSC, where the problem of transport of pollutants is only solved. In NIR, NIA, NSC, we use horizontal grid-steps of 4 km, 2 km, 500 m, accordingly. The autumn-day scenario of the atmosphere motion is considered (Figures 3 and 4).

It is suggested that at 15.00 of local time the momentary emission took place along the line AB (see Figure 1) on the height of 2500 m over land surface and this time the concentration of a pollution is equal to 100% there. It is also suggested that the drop's sizes distribution is known. The value of drop's fall velocity is calculated by Stoks's formula [4]. We consider a drops of 1 Mkm, 10 Mkm, 100 Mkm. At this numerical experiment, the solidity of a pollution is equal to 1.

Figures 5 and 6 show the distribution of the pollution concentration at 17.00 of local time. The Novosibirsk reservoir is a powerful heat island in autumn. The inhomogeneities of the underlying surface cause the forming of a compound atmospheric circulation. As the result the cloud of the pollution is torn to several parts. In Figure 5, we can see that the average concentration in the 4×4 km square grid box in the NSC does not exceed 1. There are not any local minimums or maximums of the concentration there. At the same time, in Figure 6 we see that the average concentration in the 500×500 m square grid box is larger. There are two maximums in the places, where the air mass concentrates in horizontal and lifts up. The effect of accumulation of the pollution in those places is increased owing to the settling rate decreases or vanishes due to upward air flow. The discovering of the areas, where the pollution can be concentrated for a long time without horizontal transferring, is an important problem of ecology.

## References

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