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The calculation of heating various geometries of cracks formed under pulsed heat load*

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Abstract. This paper presents a computer-aided simulation to calculate the heating of a tungsten plate with different crack geometries forming in the process of a pulsed thermal load. The results of model testing, numerical calculations and comparison with experimental data are presented. The dependence of the surface temperature on the location of cracks is shown.

1. Introduction

Currently, studying the effect of heat on the walls of the tokamak reactor is of interest. It is very important that the heat has time to go deep into the walls to the cooling tubes. The highest load is shown by impulse loads, so it is very important to study them in the sequel.

The results of heating a tungsten plate by the action of a powerful electron beam were obtained on the experimental VETA stand [1] created in BINP. It was revealed that cracks parallel to the plate surface have the strongest influence on the heat propagation. The formation of the cracks is associated with the thermal expansion of a material.

2. Statement of the problem

To calculate the temperature in a two-dimensional area containing cracks, the Fourier equation in the Cartesian coordinate system is solved:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\Big(\lambda(T)\frac{\partial T}{\partial x}\Big) + \frac{\partial}{\partial y}\Big(\lambda(T)\frac{\partial T}{\partial y}\Big), \tag{1}$$

where T is the temperature, $c(T)$ is the specific heat capacity, $\rho(T)$ is the density (Figure 1).

At the upper border of the domain, the heat flux $(\overline{n}, \nabla T)|_\gamma = \dfrac{W(x)}{\lambda}$, where $W(x)$ is the power of the heat flux, $\lambda(T)$ is the thermal conductivity. At the side and bottom borders $(\overline{n}, \nabla T)|_\gamma = 0$.

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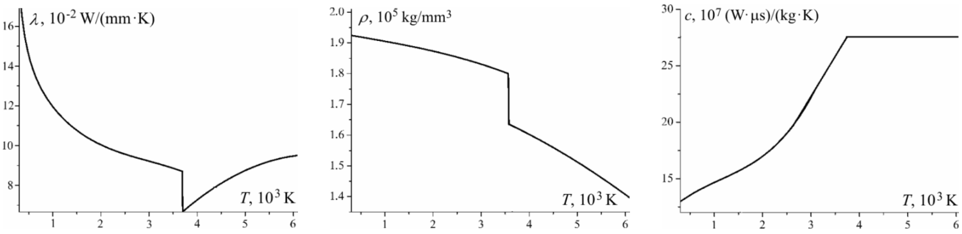


a b c

Figure 1. Graphs of the temperature dependence of density (a), thermal conductivity (b), and specific heat (c)

The crack geometry is defined at the initial time and is considered to be unchanged throughout the calculation. This is due to peculiarities of the formulation of the practical problem. Cracks are defined similarly to the boundaries of the region with the boundary condition $(\overline{n}, \nabla T)|_\gamma = 0$. If necessary, one can add the heat from the vertical cracks near to the surface of the plate. We consider that the heat does not pass through cracks as at characteristic times of calculations it can be neglected.

It is more convenient to proceed in non-dimensional variables, for example, as follows:

$$r^* = \frac{r}{r_0}, \quad \lambda^* = \frac{\lambda}{\lambda_0}, \quad \rho^* = \frac{\rho}{\rho_0}, \quad c^* = \frac{c}{c_0},$$
$$t^* = \frac{t}{t_0}, \quad T^* = \frac{T}{T_0}, \quad W^* = \frac{\lambda_0 T_0 W}{r_0}.$$

The numerical values of the parameters are given in the table:

Parameter	Typical value	Units
x, y	10^{-2}	mm
t	1	µs
T	10^3	K
c	10^7	(W · µs)/(kg · K)
ρ	10^{-5}	kg/mm^3
W	10^3	W/mm^2
λ	10^{-2}	W/(mm · K)

Equation (1) describes the Stefan problem with a free boundary [2]. The free boundary is given by discontinuous coefficients and discontinuous functions in the boundary condition describing heating [3]. These functions have discontinuities or lose smoothness at the melting point $T_m = 3695$ K.

3. Method of solution

The numerical implementation is based on the Douglas–Rachford scheme and the run method [4]. A uniform rectangular grid of nodes (x_i, y_j) is introduced in the two-dimensional solution domain

$$x_i = (i-1)h_x, \quad y_j = (j-1)h_y, \qquad i = 1, \dots, N, \quad j = 1, \dots, M,$$

where h_x, h_y are the grid steps, N and M are the number of grid nodes in x and y directions: $h_x = x_{\max}/(N-1)$, $h_y = y_{\max}/(M-1)$.

Introducing the grid functions by the rule $f_{ij}^n = f(x_i, y_j, t^n)$, we obtain

$$\begin{aligned}
&c_{ij}^n \rho_{ij}^n \frac{T_{ij}^{n+1/2} - T_{ij}^n}{\tau} = \Lambda_{xx}^{n+1/2} + \Lambda_{yy}^n,\\
&c_{ij}^n \rho_{ij}^n \frac{T_{ij}^{n+1} - T_{ij}^{n+1/2}}{\tau} = \Lambda_{yy}^{n+1} - \Lambda_{yy}^n,\\
&T_{1j}^{n+1/2} = \frac{W}{\lambda}, \quad T_{Nj}^{n+1/2} = T_{N-1j}^{n+1/2},\\
&T_{i1}^{n+1} = T_{i2}^{n+1}, \quad T_{iM}^{n+1} = T_{iM-1}^{n+1},
\end{aligned}$$

where

$$\begin{aligned}
\Lambda_{xx}^n &= \frac{1}{h}\Big(\lambda_{i+1/2j}\frac{T_{i+1j}^n - T_{ij}^n}{h} - \lambda_{i-1/2j}\frac{T_{ij}^n - T_{i-1j}^n}{h}\Big),\\
\Lambda_{yy}^n &= \frac{1}{h}\Big(\lambda_{ij+1/2}\frac{T_{ij+1}^n - T_{ij}^n}{h} - \lambda_{ij-1/2}\frac{T_{ij}^n - T_{ij-1}^n}{h}\Big).
\end{aligned}$$

4. Specifying the crack geometries

When heated cracks are of different geometries, let us consider the most common cases.

A simple case is a symmetric crack. The axis Y is along the line of symmetry of cracks and directed into the interior of the plate. The axis X is directed along the surface (Figure 2a). Calculations are carried out only in a quarter of the space. The conditions on such a crack are set as follows:

$$T_{i,j_c}^n = T_{i,j_c-1}^n, \quad T_{i,j_c+1}^n = T_{i,j_c+2}^n, \qquad 1 \le i \le i_c,$$

where i_c, j_c is the number of the node on which the crack passes.

If the vertical part of the crack is asymmetrically relative to the horizontal (Figure 2b), we set crack conditions

$$\begin{aligned}
&T_{i,j_c}^n = T_{i,j_c-1}^n, \quad T_{i,j_c+1}^n = T_{i,j_c+2}^n, \qquad 1 \le i \le i_c,\\
&T_{i_c,j}^n = T_{i_c-1,j}^n, \quad T_{i_c+1,j}^n = T_{i_c+2,j}^n, \qquad 1 \le j \le j_c.
\end{aligned}$$

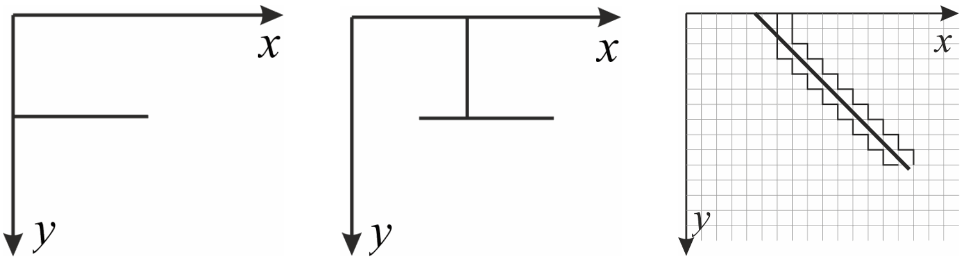


Figure 2. The crack schemes: symmetric (a), asymmetric (b), and inclined (c)

An inclined crack (Figure 2c) is set in a more complex way. We must construct an approximation on a rectangular grid. Due to the need in taking into account the boundary conditions on the upper heated surface of the plate on the axis OX, the crack is vertical at the first three nodes. The first and second nodes participate in the boundary condition at the surface, and the second and third — on the condition for the crack. Corner nodes are just two conditions: both on the vertical and on horizontal cracks.

5. Testing

A program in the Fortran language has been developed to conduct computational experiments on a pulsed tungsten heated with parallel crack surfaces. The program was tested on the quasi-one-dimensional analytical text [5] with the solution

$$T = T_0 + \frac{W\sqrt{t}}{2C_p\rho\sqrt{\chi\pi}},$$

where C_p is the thermal capacity, χ is the thermal diffusivity.

Implementation of the model with data from non-stationary and nonlinear coefficients have been investigated with various τ and h. The calculation is carried out in 0.5×0.5 mm^2 domain at a constant heating power $W = 5 \cdot 103$ W/mm^2. Figure 3 shows graphs of the surface temperature at the time instant 200 µs from the beginning of laser heating. When the grid and space steps are reduced, a relative error decreases.

The program for the complete formulation of the problem with nonlinear coefficients was tested for compliance with the calculated heating rate known from experimental data [6]. It should be noted that this formulation of the problem is characterized by a low heating depth (microns) against a relatively large heated surface (mm). Therefore, it is multi-scale problem. In the case of the model expansion, the inclusion of additional equations in the model, there will inevitably be a need for parallelization on multiprocessor computer systems.

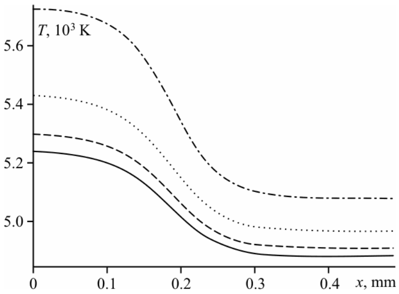
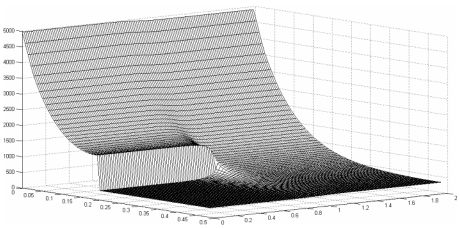


Figure 3. The surface temperature graphs for the grid parameters: $h = 1$, $\tau = 2^{-4}$ (point-dash), $h = 0.5$, $\tau = 2^{-5}$ (points), $h = 0.25$, $\tau = 2^{-6}$ (dash), $h = 0.125$, $\tau = 2^{-7}$ (straight line)

6. Results of numerical calculations

The developed program allows one to calculate the distribution of heat in the tungsten plate taking into account the heterogeneity of different geometries (Figure 4). The graphs show the temperature distributions at the cross-section of the tungsten plate heated during 200 µs at a constant power $W = 4 \cdot 10^3$ W/mm^2 for the case of a symmetric crack, during 300 µs at a constant power $W = 5 \cdot 10^3$ W/mm^2 for the case of an asymmetric crack, during 70 µs at a constant power $W = 3 \cdot 10^3$ W/mm^2 for the case of an inclined crack. The presence of microcracks increases the surface temperature, including the temperature above the melting point. Further development of the model may predict the location of cracks inside the material according to the temperature at the heated surface.

The results of computational experiments match the measurement data (Figure 5). The calculation parameters [7]: the duration of exposure of the beam 186 µs, the time of measurement 200 µs from the beginning of



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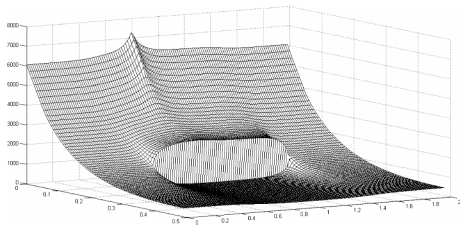

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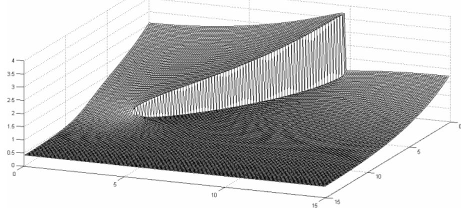
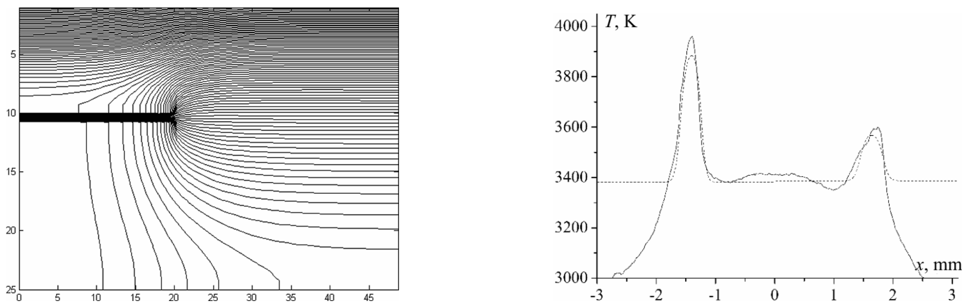


c

Figure 4. The temperature graphs around cracks: symmetric (a), asymmetric (b), and inclined (c)



a b

Figure 5. Distribution of the cross-sectional temperature (a fragment around the crack) (a) and temperature at the surface (b): experimental data (solid line) and calculated results (dotted line)

beam exposure time is 10 μs, the left crack depth of about 0.12 mm, the left crack length of about 0.2 mm, the right crack depth of about 0.15 mm, and the right crack length of about 0.145 mm. The heating was assumed to be uniform $W = 3 \cdot 10^3$ W/mm^2.

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