

## **Numerical modeling of a coherent structures ensemble with convection in the atmospheric boundary layer\***

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Some results of the numerical integration of the atmospheric hydrothermodynamics equations with the help of the LES approach are presented. It is shown that convective ensemble elements can generate macrostructures with various spatial geometries. The configuration and scales of the supercells depend on the free stream parameters.

### **1. Introduction**

At the present time, turbulence in the atmospheric boundary layer (ABL) is represented in the form of coherent structures [1]. Experimental investigations have shown [2, 3] that motions of relatively large vortices with organized structures together with chaotic isotropic turbulence are present in the atmosphere. These structures are usually caused by the development of an unstable state, as, for instance, during the penetrative convection. Convective vortices in the lower part of the ABL are realized in the form of buoyant thermals and vertical jets. Coherent structures of this type are of importance in the formation of characteristics of turbulence flow.

This is explained by the fact that the average size of thermals is much greater than that of turbulent pulsations. Therefore, a convective ensemble can hardly be represented by the Navier–Stokes equations. An effective method to investigate the convective ABL is the Large Eddy Simulation, in which “large eddies” are explicitly represented by solving the hydrothermodynamic equations, and the collective influence of vortices of the size less than the grid spacing is parametrically taken into account [4, 5].

Individual vortices form a convective ensemble with stochastic properties. This means that the generation of coherent structures and their disordered locations in space are of a random character. The ensemble elements interact with each other and with a free stream. This can bring about self-organization into large-scale conglomerates of various types. The conglomerates may be considered as coherent structures of the next hierarchical

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level. The spatial structure of such formations has often the form of "open" or "closed" supercells, extended cloud layers, and flattened out cells oriented across the flow [6].

An attempt to simulate and explain the mechanism of formation of quasiordered structures with an internal infrastructure was made in [7]. This paper is dealt with investigation of the spatial structure of convective ensembles in the ABL with the use of the LES approach.

## 2. Mathematical formulation of the problem

In the convective ABL, there are three main types of interaction, which are important in terms of energy, processes of different scales: a) the ordered mean motions; b) coherent structures ("large eddies") with characteristic size, comparable to the boundary layer thickness; c) the occurrence of the isotropic small-scale (subgrid) turbulence.

Let us introduce the rectangular Cartesian coordinate system  $(x, y, z)$ , where the axis  $z$  is directed vertically upward, and the level  $z = 0$  coincides with the Earth's surface. Let us present the vector-function  $\phi = (u, v, w, \theta, \pi)$ , where  $u, v, w$  are the velocity vector components along the axes  $x, y, z$ ;  $\theta$  is potential temperature; and  $\pi$  is an analogue to pressure. Let us seek it in the form of the sum

$$\phi = \Phi + \phi', \quad (1)$$

where the fields  $\Phi(z, t) = (U, V, 0, \Theta, \Pi)$  and  $\phi' = (u', v', w', \theta', \pi')$  describe the processes a) and b) respectively.

Let  $L_x, L_y$  stand for the horizontal size of the domain, where the non-stationary penetrating convection as an ensemble of spontaneously forming thermals is generated, and let us assume the periodicity of the processes along  $x, y$ .

Substituting representation (1) into equations of mesoscale atmospheric dynamics [9], let us average them in the horizontal plane. As a result we arrive at the system of equations, describing the mean current in the ABL

$$\begin{aligned} \frac{\partial U}{\partial t} &= l(V - V_G) + \frac{\partial}{\partial z} K \frac{\partial U}{\partial z} - \frac{\partial}{\partial z} \overline{uw}, \\ \frac{\partial V}{\partial t} &= -l(U - U_G) + \frac{\partial}{\partial z} K \frac{\partial V}{\partial z} - \frac{\partial}{\partial z} \overline{vw}, \\ \frac{\partial \Theta}{\partial t} &= \frac{\partial}{\partial z} K_T \frac{\partial \Theta}{\partial z} - \frac{\partial}{\partial z} \overline{\theta w}, \end{aligned} \quad (2)$$

where  $U_G, V_G$  are the geostrophic wind components,  $l$  is the Coriolis parameter,  $K$  is vertical turbulent exchange coefficient of the subgrid scale,

$K_T = K/\text{Pr}$ ,  $\text{Pr}$  is the Prandtl number in the ABL. Here and below the primes at convective deviations are omitted, feature above variables means horizontal average.

Let us simulate the mean current subgrid-scale turbulence in the ABL on the basis of equations of the semi-empirical turbulence theory for  $b, \epsilon$ -equations [5].

The system of equations for the description of the mesoscale convective processes b) in the ABL is obtained by the component-by-component subtraction (2) from the respective equations of the original system

$$\begin{aligned} \frac{du}{dt} + w \frac{\partial U}{\partial z} &= -\frac{\partial \pi}{\partial x} + lv + D_{xy}u + \frac{\partial}{\partial z} K \frac{\partial u}{\partial z} + \frac{\partial}{\partial z} \overline{uw}, \\ \frac{dv}{dt} + w \frac{\partial V}{\partial z} &= -\frac{\partial \pi}{\partial y} - lu + D_{xy}v + \frac{\partial}{\partial z} K \frac{\partial v}{\partial z} + \frac{\partial}{\partial z} \overline{vw}, \\ \frac{dw}{dt} &= -\frac{\partial \pi}{\partial z} + \lambda \theta + D_{xy}w + \frac{\partial}{\partial z} K \frac{\partial w}{\partial z}, \\ \frac{d\theta}{dt} + w \frac{\partial \Theta}{\partial z} &= D_{xy}\theta + \frac{\partial}{\partial z} K_T \frac{\partial \theta}{\partial z} + \frac{\partial}{\partial z} \overline{w\theta}, \\ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \end{aligned} \quad (3)$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + (U + u) \frac{\partial}{\partial x} + (V + v) \frac{\partial}{\partial y} + w \frac{\partial}{\partial z}$$

is an individual derivative operator,

$$D_{xy} = \frac{\partial}{\partial x} K_x \frac{\partial}{\partial x} + \frac{\partial}{\partial y} K_y \frac{\partial}{\partial y},$$

$K_x, K_y$  are the lateral turbulence coefficients of subgrid scale,  $\lambda$  is the buoyancy parameter.

Let us consider the boundary and the initial conditions. We define the vertical structure of the domain in the following way:  $H$  is the upper boundary of the ABL,  $h$  is the air layer thickness, for which the assumption of constant turbulent fluxes is valid.

For the mean current equations in the ABL (2) let us set the following conditions:

$$U = U_G, \quad V = V_G, \quad \frac{\partial \Theta}{\partial z} = \gamma_H \quad \text{at } z = H; \quad (4)$$

$$K \frac{\partial U}{\partial z} = c_u |\vec{U}| U, \quad K \frac{\partial V}{\partial z} = c_u |\vec{V}| V, \quad -\rho_0 c_p K_T \frac{\partial \Theta}{\partial z} = Q_T \quad \text{at } z = h; \quad (5)$$

where  $\gamma_H$  is standard stratification of the free atmosphere,  $c_u$  is resistance coefficient,  $Q_T$  is a heat flux from the surface to the atmosphere,  $c_p$  is specific heat of the air at constant pressure,  $\rho_0$  is the air density.

The boundary conditions for system (3) are the following:

$$u = v = w = 0, \quad \theta = 0 \quad \text{at } z = H; \quad (6)$$

$$u = v = w = 0, \quad \theta = \theta_0(t, x, y) \quad \text{at } z = h; \quad (7)$$

where  $\theta_0$  is random low-amplitude temperature perturbation.

The following initial condition was set for (2):

$$\Phi = \Phi_0 \quad \text{at } t = t_0, \quad (8)$$

where  $\Phi_0$  is stationary solution to system (2) in the absence of convection and  $Q_T = 0$ .

The formulated problem (2)–(8) was solved by an implicit finite difference method based on a version of the splitting method. Equations (3) were discretized in terms of the initial variables “velocity–pressure”. The numerical algorithm includes the transfer and the turbulent exchange stages in each of the directions  $x$ ,  $y$ ,  $z$  and the correction stage, providing dynamic conformity of the fields and an increase in accuracy of the scheme by recalculation of nonlinear terms. The scheme has the second accuracy order in all the variables and is stable within the range of admissible values of physical parameters. The computational grid consists of  $64 \times 64$  cells at  $L_x = L_y = 10$  km. The time step  $\Delta t$  equals to 6 seconds.

### 3. Calculation results

The main external parameters of the problem are the geostrophic wind speed and the heat flux from the underlying surface. The characteristic value of  $Q_T$  in the unstable surface layer is  $10 \text{ W/m}^2$  [8]. In accordance with this, let us consider four versions of the calculations with the following sets of parameters:

1.  $U_G = 0$ ,  $Q_T = 5 \text{ W m}^{-2}$ ;
2.  $U_G = 0$ ,  $Q_T = 15 \text{ W m}^{-2}$ ;
3.  $U_G = 5 \text{ m s}^{-1}$ ,  $Q_T = 5 \text{ W m}^{-2}$ ;
4.  $U_G = 10 \text{ m s}^{-1}$ ,  $Q_T = 15 \text{ W m}^{-2}$ .

In experiment 1, a relatively small heat flux under calm conditions is specified.

Isolines of the horizontal cross-sections of the field  $w$  at a level of the kinetic energy maximum,  $z = 350$  m, are shown in Figure 1 (field 1). It can be seen that about 120 convective elements are formed in the region. The maximum velocity of updrafts was  $1.5 \text{ m/s}$ , and the maximum velocity of

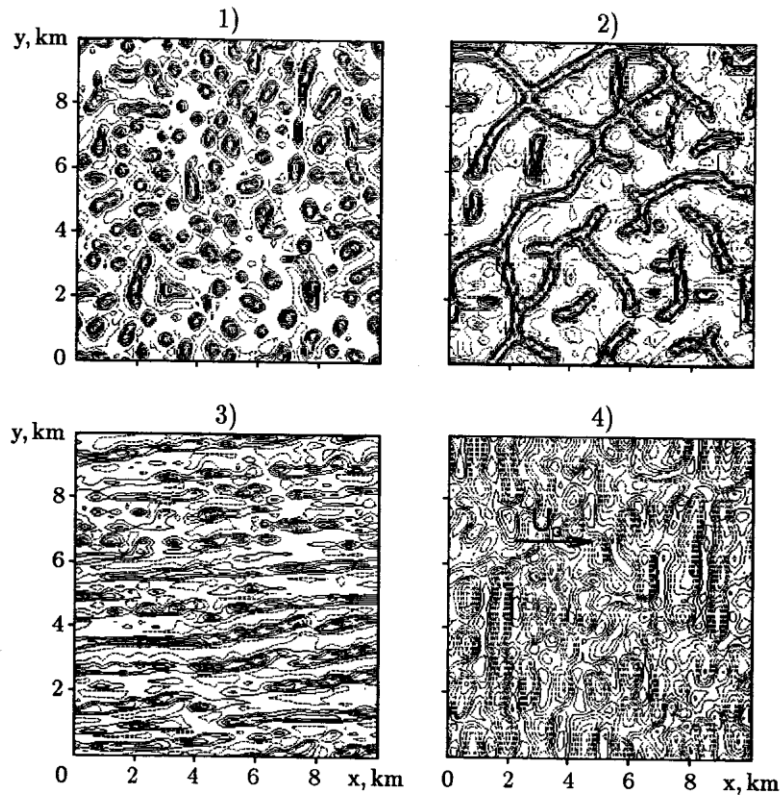
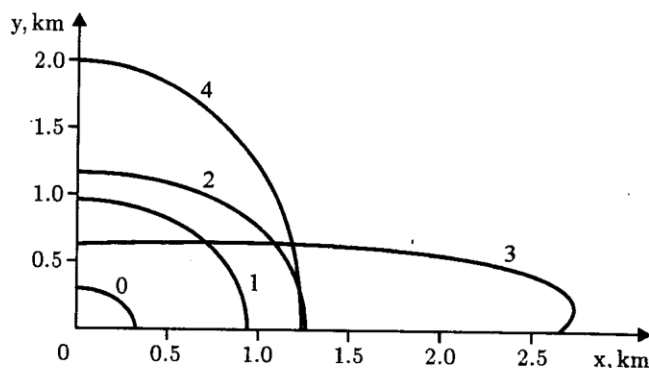


Figure 1. Horizontal sections of the field  $w$  in numerical experiments 1, 2, 3, 4

downdrafts – 0.54 m/s. The interval of isoline values is 0.25 m/s (formations with positive  $w$  can be easily identified in Figure 1). Note that the distribution of thermals in the region is uniform. Curve 1 in Figure 2 is constructed for the mean energy wave number. It shows a typical configuration and scale of coherent structures in the plane  $(x, y)$ . The average horizontal size of convective elements is 900 m. According to the observational data [2], the horizontal size of thermals vary from 150 to 1000 m and over, and their spatial period is in the range between 500–700 m. Thus, the results of numerical calculations turned out to be very close to natural data.

An increase in  $Q_T$  (variant 2) causes an increase in the average size of the convective structures (Figure 1, field 2) and a decline in their number. The typical horizontal sizes are the same in all the directions and equal to 1200 m (Figure 2, curve 2). Small thermals coagulate into conglomerates of irregular shapes with maximum  $w = 2.1$  m/s (min  $w = -0.8$  m/s). The coherent structure of the regions with updrafts is qualitatively similar to aggregated cloud systems of the open type. Observations show [6] that narrow updrafts that are organized into the large-scale supercells develop in such systems.



**Figure 2.** The distribution of the mean size of convective structures in the planes  $x, y$  in experiments 1, 2, 3, 4. The curve 0 characterizes the scales of random disturbances fields  $\theta_0(x, y)$

The free stream (geostrophic) velocity is specified in variant 3. As a result of advective transport, the  $w$ -field acquires a quasi-two-dimensional form with trails stretched upwind. The convective wave "trains" (Figure 1, field 3) are located along these trails. The horizontal dimensions considerably increase in the direction of transport, and an infrastructure in the form of a system of shape-preserving thermals with the long-wave modulation is formed in the convective trains. In course of time, small thermals will merge as a result of coagulation, forming a large vortex with longitudinal orientation. A cascade process of enlargement of coherent structures in the free stream was observed during experimental investigations of the ABL with the help of an aircraft laboratory [3]. The spread of vortex trails in width along the axis  $y$  is not great, the average width being 600 m. This can be seen in Figure 2 (curve 3), which shows that the horizontal scales maximum of convective elements is obtained in the direction of the axis  $x$ .

Let us consider the structure of the field  $w$  obtained in variant 4 at a strong wind and rather a high value of heating of the surface layer. The flow pattern is shown in Figure 1 (field 4). Convective rolls transverse to the flow with alternating single thermal jets are clearly seen. This is also illustrated by Figure 2, curve 4: the longitudinal size of perturbations is 1250 m, and their transverse size is about 2000 m. The possibility of generation of structures with transverse orientation of convective cells is confirmed by observations: see snapshots of cloud fields, which are qualitatively similar to the structure of field 4 in Figure 1 [10].

From the above analysis of variants 1–4 we can conclude that the main criterion of formation of quasi-two-dimensional convective rolls is the ratio between the free stream velocity and the heat source strength. At small heat fluxes and a moderate velocity, the convective structures are stretched upwind. As instability of the surface layer increases, their longitudinal size decreases, and the transverse size increases.

## 4. Conclusion

The above presented LES-model makes it possible to describe a convective ensemble of a totality of quasi-ordered coherent structures ("large eddies"). The possibility of formation of convective clusters, i.e., elementary structures aggregated into coherently stable systems with various spatial organizations, has been theoretically proved. In terms of the quasi-ordering and the quasi-determinacy, such systems can be considered as coherent structures of a hierarchical level that is higher than that of individual thermals. It has been shown that the calculated fields are in qualitative (and, for some variants, quantitative) agreement with phenomena observed in the convective ABL.

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