Bull. Nov. Comp. Center, Math. Model. in Geoph., 22 (2020), 41–50

Modeling the inhomogeneous two-phase flows with an admixture in inclined channels*

Yu. Perepechko, K. Sorokin, Sh. Imomnazarov

Abstract. The paper studies the flow of a two-phase medium in a gravity field channel. The thermodynamically consistent equations of the mathematical model of the dynamics of a two-velocity medium with an admixture were developed within the framework of the method of conservation laws. Its numerical implementation was carried out based on the control volume method. The nature of the motion of inhomogeneous flows in horizontal and inclined channels, the development of flow instability, as well as the effect of the surface tension gradient on flow regimes are investigated.

Keywords: suspension, emulsion, admixture, two-velocity hydrodynamics, method of conservation laws, method of control volume.

Introduction

Modeling the flows of heterophase condensed media is currently central due to a wide range of applications in solving problems arising in the study of both natural and technological systems. A correct hydrodynamic analysis of heterophase flows of various mixtures of viscous liquids, suspensions of solid particles, especially, the analysis of the development of various kinds of flow instabilities, in particular, convective and sedimentation instability, must be based on the consistent theory of the two-velocity hydrodynamics. Such a phenomenological approach that ensures the physical correctness of the model is the method of thermodynamically consistent conservation laws [1–4]. Within the framework of this approach, the dynamics of two-velocity media was investigated, including saturated porous media containing impurities [5], suspensions of solid particles in a melt [6]. In this study, based on this method, the dynamics of a mixture of mobile media is investigated in a two-velocity hydrodynamic approximation in the presence of an impurity and taking into account the surface tension of the dispersed phase; various modes of a homogeneous and an inhomogeneous two-phase flow in a pipe with different inclination angles in a gravity field are studied. The used hydrodynamic model of dispersed media takes into account dissipative (thermal and viscous effects, diffusion, interfacial friction) and surface phenomena in a two-phase medium.

*Supported by the Russian Scientific Foundation under Grant 20-19-00058.

1. Mathematical model

The present study deals with the study of the nonlinear non-stationary dynamics of two-velocity condensed media, such as suspensions, emulsions, colloidal solutions. The elementary volume of such a heterophase medium is characterized by the partial densities ρ_1, ρ_2 and the velocities $\boldsymbol{u}$, $\boldsymbol{v}$ of the dispersed and dispersing phases, the density of the impurity, if any, the number of particles of the dispersed phase n, as well as the temperature T and the concentration of the impurity c. The governing equations for the dynamics of a heterophase medium are derived using the method of conservation laws under the assumption of phase equilibrium with respect to temperature and pressure [6, 7]. In the dissipative case in a gravity field, the governing equations can be represented in the following form

$$\frac{\partial \rho_1}{\partial t} + \operatorname{div}(\rho_1 \boldsymbol{u}_1) = 0, \tag{1}$$

$$\frac{\partial \rho_2}{\partial t} + \operatorname{div}(\rho_2 \boldsymbol{u}_2) = 0, \tag{2}$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(n \boldsymbol{u}_1) = 0, \tag{3}$$

$$\frac{\partial \rho_a}{\partial t} + \operatorname{div}\left(c_1\rho_1\boldsymbol{u}_1 + c_2\rho_2\boldsymbol{u}_2 - \nu\frac{1}{T}\nabla T - DT\nabla\frac{\mu_a}{T}\right) = 0, \tag{4}$$

$$\begin{aligned}\frac{\partial j_i}{\partial t} + \partial_k\big(\rho_1 u_{1i}u_{1k} + \rho_2 u_{2i}u_{2k} + p\delta_{ik} - (\eta_1+\eta_{12})u_{1ik} - \\ (\eta_2+\eta_{12})u_{2ik}\big) = \rho\mathbf{g},\end{aligned} \tag{5}$$

$$\begin{aligned}\frac{\partial u_{2i}}{\partial t} + (\boldsymbol{u}_2,\nabla)u_{2i} = -\frac{1}{\rho}\partial_i p + \frac{n}{\rho}\varsigma\partial_i\sigma + \frac{\rho_1}{2\rho}\partial_i\boldsymbol{w}^2 + bw_i + \\ \frac{1}{\rho_2}\partial_k(\eta_2 u_{2ik} + \eta_{12}u_{1ik}) + g_i,\end{aligned} \tag{6}$$

$$\frac{\partial S}{\partial t} + \operatorname{div}\left(S_1\boldsymbol{u}_1 + S_2\boldsymbol{u}_2 - (\kappa+\nu\mu_a)\frac{1}{T^2}\nabla T + (D\mu_a - \nu T)\nabla\frac{\mu_a}{T}\right) = \frac{1}{T}R, \tag{7}$$

$$\begin{aligned}R = \rho_2 b\boldsymbol{w}^2 + \kappa\left(\frac{\nabla T}{T}\right)^2 + 2\nu\nabla\frac{\mu_a}{T}\nabla T + DT^2\left(\nabla\frac{\mu_a}{T}\right)^2 + 2\lambda_1\frac{\nabla T}{T}\boldsymbol{w} + \\ 2\lambda_2\nabla\left(\frac{\mu_a}{T}\right)\boldsymbol{w} + \frac{1}{2}\eta_1 u_{1ik}u_{1ik} + \frac{1}{2}\eta_2 u_{2ik}u_{2ik} + \eta_{12}u_{1ik}u_{2ik}.\end{aligned} \tag{8}$$

It is defined above that $\rho = \rho_1 + \rho_2$, $\boldsymbol{j} = \rho_1\boldsymbol{u} + \rho_2\boldsymbol{v}$ are the density and momentum of a two-velocity medium; p is the pressure; μ_a is the chemical potentials of a two-phase medium and impurities; σ is the surface tension tensor; ς is the specific surface area of the dispersed phase; $\mathbf{g}$ is the acceleration of gravity. The impurity concentration in a two-phase medium and in phases is determined by the relations $c = \rho_a/\rho$, $c_1 = c + 2\lambda_1\rho_2/\rho_1$, $c_2 = c - 2\lambda_1$. S is the entropy of a two-phase medium, $S_2 = S\rho_2/\rho - 2\lambda\rho_2/\rho$, $S_1 = S\rho_1/\rho + 2\lambda\rho_2/\rho$ are the entropy of phases. The kinetic coefficients of the interfacial friction b, the shear viscosity of the phases η_i, the mutual viscosity η_{12}, the thermal conductivity of the two-phase medium κ and the coefficients λ_1, λ_2, ν are the functions of thermodynamic parameters. The strain rate tensors are determined by the relations $u_{1ik} = \partial_k u_{1i} + \partial_i u_{1k} - 2/3\delta_{ik}\operatorname{div}\boldsymbol{u}_1$, $u_{2ik} = \partial_k u_{2i} + \partial_i u_{2k} - 2/3\delta_{ik}\operatorname{div}\boldsymbol{u}_2$. The bulk viscosity effects are not considered in the model.

The equations of state of a two-phase medium, which close the dynamic equations (1)–(7), are obtained in the linear approximation: $\delta\rho = \rho\alpha\,\delta p - \rho\beta\,\delta T$, $\delta s = c_p\,\delta T/T - \beta\,\delta p/\rho$. The coefficients of the volumetric compression α, the thermal expansion β, and the specific heat c_p are additive in phases. The impurity is taken into account in the approximation of an ideal solution: $\mu_a = d_1 p + d_2 T + \bar{R}T\ln c$, $\bar{R}$ is the universal gas constant. The surface tension is determined by the Shishkovsky ratio $\sigma = \sigma_0(T_c - T)/(T_c - T_{\text{ref}}) - \sigma_1\ln(1 + ac)$.

2. Numerical model

The difference approximation of the equations of the two-velocity hydrodynamics is based on the control volume method [8, 9], which provides accurate integral conservation of such quantities as mass, momentum and energy in any volume. Discretization of the governing equations was carried out on a rectangular uniform grid with a shift of the design nodes for the components of the velocity vectors with respect to the design nodes for the remaining variables. A completely implicit scheme with respect to time is used. When approximating the convective terms for calculating fluxes through the faces of control volumes, the second order HLPA scheme was implemented [10], which provides a good accuracy and satisfies the convective boundedness criterion. The diffusion terms are approximated using a central difference scheme. To calculate the pressure field consistent with the flow field, an analogue of the IPSA iterative procedure was implemented [11]. The approximation of the terms that determine the force interaction of the phases is carried out completely implicitly. Within the framework of the developed computational algorithm, the continuity equation is not resolved explicitly; its discrete analogue is used to derive discrete analogues of other equations and to derive a correction equation for pressure. The finite difference approximation of the boundary conditions in both cases is similar and is carried out according to the second-order scheme [12].

To calculate the velocity fields and the pressure field matched to the continuity equation, a version of the iterative procedure SIMPLE was implemented [8]. When switching to a new time step, an initial assumption about the approximate value of the pressure field is made, while the true value is determined through the correction. Corrections for the components of the velocity vectors are introduced in a similar way. Further, by subtracting the exact and approximate discrete analogs of the equations of motion and eliminating the number of terms, which is allowed within the framework of using the basic version of the SIMPLE procedure, expressions for the corrections are introduced. A feature of the IPSA procedure is the use of a phase-wide discrete analog of the continuity equation when deriving a correction equation for pressure.

The step-by-step scheme for applying the procedure is as follows:

1. Start with an approximate pressure field;
2. Solve the discrete analogs of equations of motion for finding approximate values of velocity fields;
3. Find the pressure correction;
4. Calculate a new pressure value;
5. Calculate the new values of the fields of the velocity vector components;
6. Solve the discrete analogs of the remaining equations of the model, recalculate the density and temperature fields using thermodynamic relations;
7. Assign the corrected pressure field as a new one, return to Step 2 and repeat the procedure until the iterative process converges.

For the numerical solution of systems of linear algebraic equations of discrete analogs of control equations and a correction equation for pressure, the alternating direction method and the parallel direct solver PARDISO, implemented as part of the Intel MKL mathematical library [13], were used.

3. Calculation results

In the numerical modeling, the problem of the evolution of the dispersed phase in the case of the pressure flow of a heterophase medium in an inclined channel, the development of convective and sedimentation instability, as well as the effect of an admixture on the flow pattern by changing the surface tension gradient and redistribution of an admixture between phases were investigated. A mixture of viscous compressible fluids was taken as a model heterophase medium. By specifying the thermodynamic and kinetic parameters, such a model allows one to describe both emulsions and suspensions.

The computational domain is taken as a rectangular channel with 0.5×2 m size. The side walls of the channel are defined by fixed impermeable surfaces, that is, the planes serve as adiabatic insulation and there are no normal and tangential velocity components. Either a pressure drop is set between the inlet and outlet boundaries of the region, or the constant phase velocities are set at the inlet boundary. For the temperature of the medium and the impurity concentration, the Dirichlet conditions at the inlet boundary and the Neumann conditions at the outlet boundary are set. As the initial distribution of particles of the dispersed phase, a layered distribution, that is inhomogeneous over the channel cross-section, was specified. This is because a uniform initial distribution of the phase content, sedimentation processes are sufficiently weak, and instability does not develop or, depending on the specified pressure drop, develops slowly, the flow is almost stationary. It should be noted that the distribution of physical and partial phase densities at the initial instant of time is set consistently with setting the pressure as a result of the iterative process with allowance for the gravity field.

The values of physical parameters for the dispersed phase are: $\rho_s^f = 880$ kg/m^3, $\alpha_s = 1.2 \cdot 10^{-10}$ Pa^{-1}, $\eta_s = 0.1$ kg/(m s); for the dispersion phase: $\rho_l^f = 998$ kg/m^3, $\alpha_l = 4.7 \cdot 10^{-9}$ Pa^{-1}, $\eta_l = 0.001$ kg/(m s). The volume fraction of the dispersion phase at the initial time is equal to $\phi = 0.5$. In addition, the following parameters were set: $d_1 = 0.1$ m^3/kg, $d_2 = 0.001$ m^2/(K s^2), $T_c = 513$ K, $T_{\mathrm{ref}} = 293$ K, $a_1 = 7 \cdot 10^{-2}$ N/m, $\sigma_2 = 0.1 \div 2$ N/m. The parameter σ_2 characterizes the value of the derivative of the surface tension from the concentration of the impurity in the solution and determines its surface activity. In the calculations, the values of the dissipative parameters $\lambda_2 = 10^{-2}$ kg/(m s^2), $\lambda_1 = 10^{-6}$ and the diffusion coefficient $D = 2 \cdot 10^{-9}$ m^2/s were used. The calculations were made with allowance for a change in the friction coefficient associated with a change in the density of the dispersed phase during the dynamic process of phase redistribution.

The figures below show the distributions of the density of the number of particles of the dispersed phase in the two-phase medium n and the relative velocity of movement of the dispersed and dispersive phases $\boldsymbol{w}$.

Figure 1 shows the results of modeling a flow in a horizontal channel in a gravity field. At the inlet boundary of the channel, the horizontal components of the vectors of velocities of the carrier and dispersed phases are specified, respectively, $u_{1x} = u_{2x} = 0.1$ m/s. At the initial moment of time, there is no phase movement; from the upper to the lower boundary of the computational domain, the profile of the content of the dispersion phase is set from 0.8 up to 0.2 with rather a sharp change in the volume content of phases identified along the central axis of the channel.

The formation of a zone of development of instability is observed, which reaches 70–80 cm. Further along the flow, in the region of an increased gradient of the volumetric content of phases, fluctuations rapidly grow and

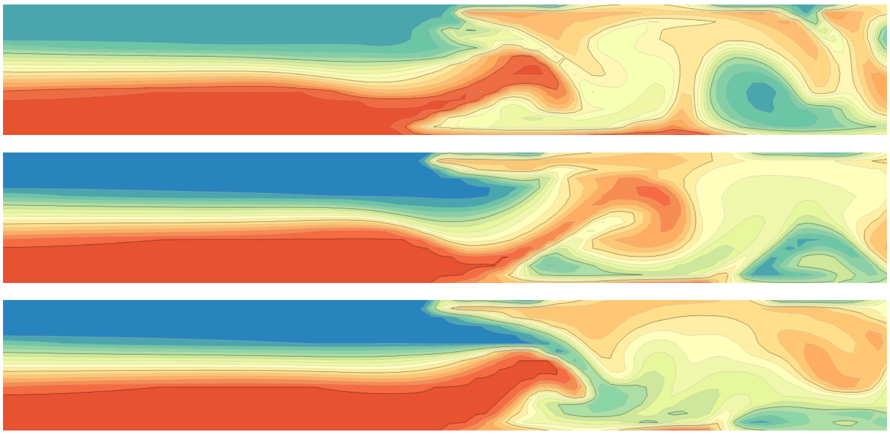

Figure 1. Distribution of the specific content of dispersed phase particles in a horizontal channel for the times of 9, 13.5, and 15.6 s

a strongly mixed flow is formed with a displacement of the dispersed phase towards the channel boundary. In the future, this picture of the development of instability and the flow of the dispersed phase is constantly reproduced.

The simulation results of the flow in an inclined channel with an inclination angle of 5° and 25° are shown in Figure 2 and 3. At the inlet boundary of the channel, the horizontal components of the vectors of velocities of the carrier and dispersed phases are respectively, set as $u_{1x} = u_{2x} = 0.1$ m/s. At the initial instant of time from the upper to the lower boundary of the computational domain, a drop is set. The distribution of the content of the dispersed phase was set similar to the problem without a channel tilt.

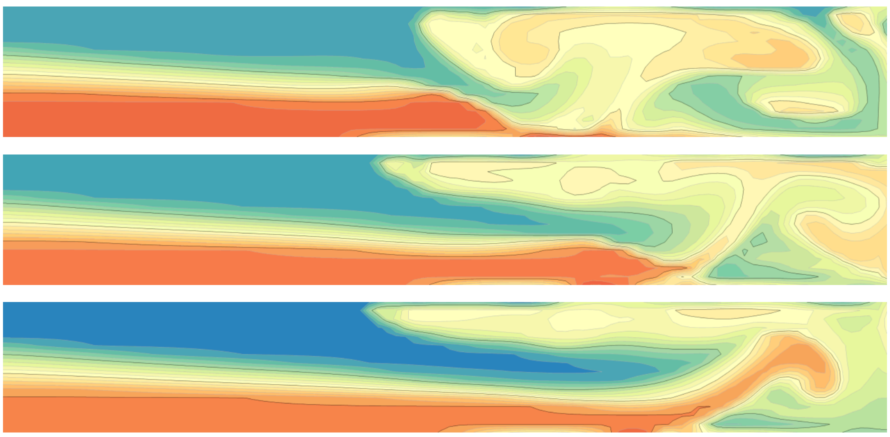

Figure 2. Distribution of the specific content of dispersed phase particles in an inclined channel with an inclination angle of 5° for the times of 9, 13.5, and 15.6 s

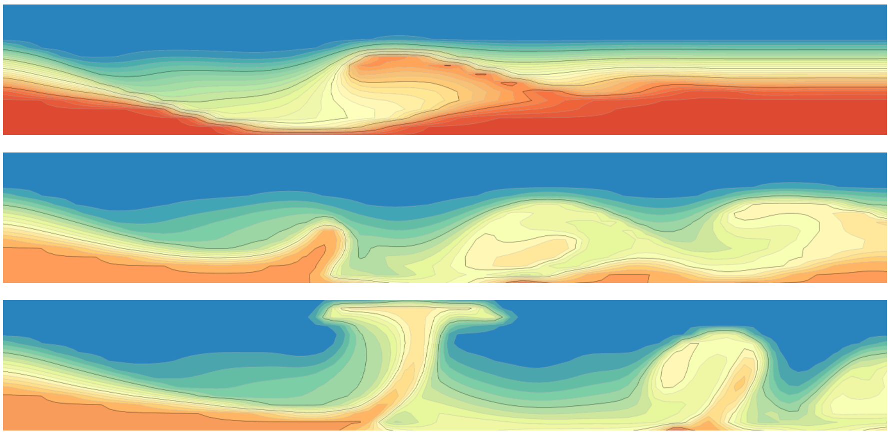

Figure 3. Distribution of the specific content of dispersed phase particles in an inclined channel with an inclination angle of 25° for the times of 9, 13.5, and 15.6 s

The development of the sedimentation instability against the hydrodynamic background in this formulation of the problem with a gap inclination relative to the gravity field of 5° and, especially, 25° demonstrates a rather a strong difference from the development of instability in the problem for a flow in a horizontal channel. The zone of the initial development of instability is also present, however, the formation of vortex structures is observed throughout the entire region of the flow of the water-oil mixture. Further over the downstream, the initial stratification of the two-phase mixture is retained, and the region of a highly mixed flow is shifted to the lower part of the channel.

It should be noted that the development of instability in a two-phase flow in this formulation of the problem significantly differs from the problem with a given pressure drop. When a constant flow rate is set, the instability development zone of the order of 40 cm is observed at the inlet boundary of the channel. Further along the flow, in the region of an increased phase content gradient, fluctuations rapidly grow and a strongly mixed flow is formed with the evolution of the dispersed phase towards the channel boundary. This picture of the course and development of instability is constantly reproduced. When a pressure drop is specified, the instability develops in the entire region of the channel. An increase in the pressure drop leads to an active drift of convective structures and further the downstream. The mixture composition becomes almost uniform due to mixing throughout the channel cross-section.

Figures 4–6 show the effect of an impurity on the flow of a heterophase medium due to the redistribution of an impurity between the phases (Figure 4) and due to a change in the surface tension (Figures 5, 6).

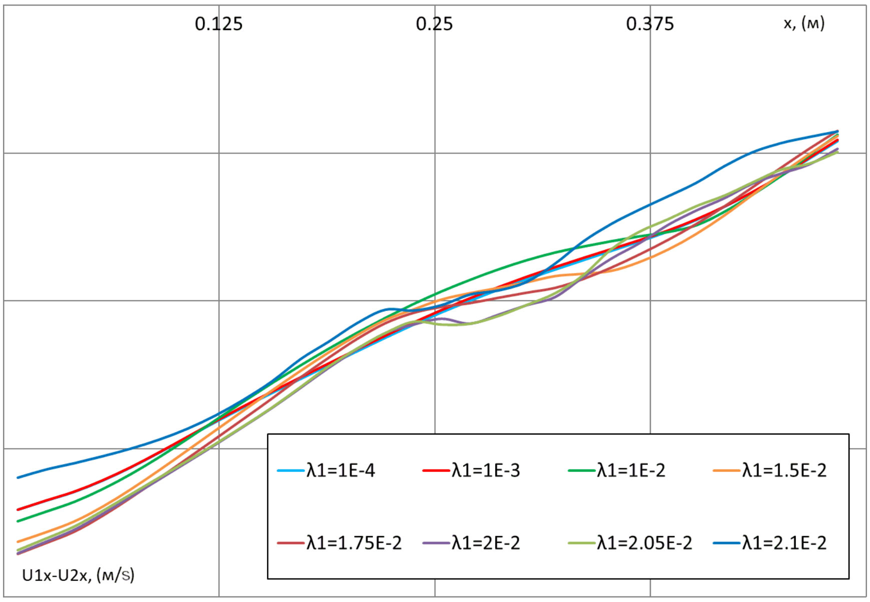

Figure 4. Profiles along the central cross-section of the computational domain when varying the parameter $\lambda_1 = 0.01, 0.1, 1, 1.5, 1.75, 2, 2.05, 2.1 \cdot 10^{-2}$

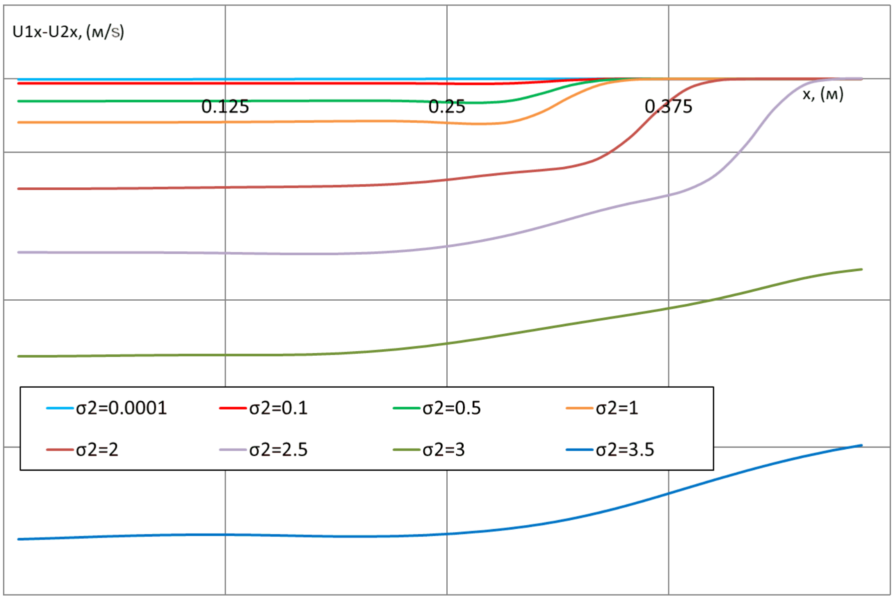

Figure 5. Profiles along the central cross-section of the computational domain when varying the magnitude of the surface tension gradient for the values $\sigma_2 = 10^{-4}, 0.1, 0.5, 1, 2, 2.5, 3, 3.5$ N/m

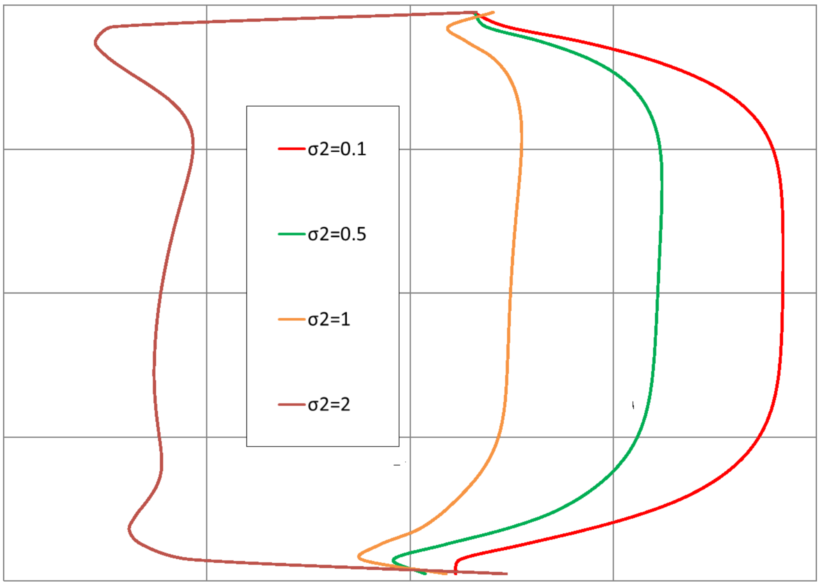

Figure 6. Profiles of the central vertical cross-section of the computational domain when varying the magnitude of the surface tension gradient for the values $\sigma_2 = 10^{-4}$, 0.1, 0.5, 1, 2, 2.5, 3, 3.5 N/m

The results obtained show a weak influence of the effect associated with different rates of entrainment of impurities by phases on the flow regime of the heterophase mixture (see Figure 4). At the values of the parameter $\lambda_1 \leq 10^{-3}$, the presence of an impurity does not affect the flow regime; at the value $\lambda_1 > 2 \cdot 10^{-2}$, the effect on the flow regime becomes noticeable.

The effect of the impurity on the flow regime of the dispersed mixture due to a change in the value of the surface tension appears to be more significant. At $\sigma_0 = 7 \cdot 10^{-2}$ N/m, for the values $\sigma_2 > 1$ N/m, a significant effect on the flow regime is observed (see Figures 5, 6).

Conclusion

In this paper, based on the control volume method, the nonlinear processes of heat and mass transfer and diffusion in heterophase condensed media in the presence of an impurity and with allowance for the surface tension of the dispersed phase is studied. The development of instability of an inhomogeneous heterophase flow in inclined channels is considered. The effect of an impurity on the dynamics of the motion of such media, which leads to the acceleration of the motion of the dispersion phase and a significant deceleration of the motion of particles of the dispersed phase in an unsteady flow regime, is shown.

References

[1] Khalatnikov I.M. An Introduction to the Theory of Superfluidity. — New York: Benjamin, 1965.

[2] Roberts P.H., Loper D.E. Dynamical processes in slurries // Structure and Dynamics of Partially Solidified Systems / NATO ASI Series. — 1987. — Vol. 125. — P. 229–290.

[3] Dorovsky V.N. Mathematical models of two-velocity media. Part I // Math. Comput. Model. — 1995. — Vol. 21, No. 7. — P. 17–28.

[4] Dorovsky V.N., Perepechko Y.V. Mathematical models of two-velocity media. Part II // Math. Comput. Model. — 1996. — Vol. 24, No. 10. — P. 69–80.

[5] Dorovsky V.N., Perepechko Y.V. A hydrodynamic model for a solution in fracture-porous media // Russian Geology and Geophysics. — 1996. — Vol. 37, No. 9. — P. 117–128.

[6] Dorovsky V.N. The hydrodynamics of particles suspended in a melt with the self-consistent concentration field of an admixture I // Computers and Mathematics with Applications. — 1998. — Vol. 35, No. 11. — P. 27–37.

[7] Dorovsky V., Perepechko Yu., Sorokin K. Two-velocity flow containing surfactant // J. Engineering Thermophysics. — 2017. — Vol. 26. — P. 160–182.

[8] Patankar S. Numerical Heat Transfer and Fluid Flow. — CRC Press, 1980.

[9] Perepechko Yu.V., Sorokin K.E., Imomnazarov Kh.Kh. Modeling of the nonlinear motion of saturated granular media // Bull. Novosibirsk Comp. Center. Ser. Num. Mod. in Atm., Ocean and Env. Studies. — Novosibirsk, 2016. — Iss. 15. — P. 51–65.

[10] Zho J.A. low-diffusive and oscillation free convection scheme // Communications in Applied Numerical Methods. — 1991. — Vol. 7. — P. 225–232.

[11] Darwish M., Moukalled F. A unified formulation of the segregated class of algorithms for multifluid flow at all speeds // Numerical Heat Transfer. Part B. — 2001. — Vol. 40. — P. 99–137.

[12] Hayase T., Humphrey J.A.C., Greif R. A consistently formulated QUICK scheme for fast and stable convergence using finite-volume iterative calculation procedure // J. Comput. Phys. — 1992. — Vol. 98. — P. 108–118.

[13] Schenk O., Gartner K. Solving unsymmetric sparse systems of linear equations with PARDISO // J. Future Generation Computer Systems. — 2004. — Vol. 20, No. 3. — P. 475–487.