Letter to Editorial Board

on the book of A.Yu. Bezhaev and V.A. Vasilenko "Variational Spline Theory"

I have found in [1] some omissions:

1. Formula

$$S = A^*(AA^*)^{-1}A \tag{1.47}$$

(see Section 1.4.2) for the representation of the interpolating spline operator S is valid not only on the subspace $N(T)_*^{\perp}$, but on the whole space X too. This follows from the equivalence of the spline interpolation problem

$$\sigma = \arg \min_{x \in A^{-1}(A\varphi_*)} \|Tx\|^2$$

and the problem

$$\sigma = \arg \min_{x \in A^{-1}(A\varphi_*)} \|Tx\|^2 + \|\tilde{A}x\|^2,$$

where $\tilde{A} : X \to \tilde{Z}$ is *any* bounded operator satisfying the conditions: $N(\tilde{A}) \supset N(A)$, and $(T, \tilde{A})$ is a spline-pair (cf. [2]).

Hence, in the formulation of Theorem 4.1 (Section 4.1.2), we may replace the subspace $N(T)_*^{\perp}$ by X.

2. The formulation of Theorem 4.1 is incorrect. The *weak* convergence of the subspaces E_τ must be replaced by the *strong* one.

In fact, in the proof of this theorem the property

$$\forall x, y \in N(T)_*^{\perp} \qquad (B_\tau x, y)_* \to (x, y)_*, \quad \tau \to 0 \tag{4.31}$$

is used (here B_τ is the ortoprojector onto the subspace E_τ in the norm $\|\cdot\|_*$). However, this property is nothing but the strong convergence of the subspaces E_τ. Indeed, the ortoprojector B_τ satisfies the condition

$$(B_\tau x - x, B_\tau x)_* = 0.$$

So, substracting this identity from (4.31) and substituting $y = x$ we have

$$(B_\tau x - x, B_\tau x - x)_* \to 0.$$

3. Formula

$$S_\alpha = A^*(\alpha I + AA^*)^{-1}A \tag{1.52}$$

(see Section 1.4.3) for the representation of the smoothing spline operator S_α is valid only in the case of *natural spline smoothing*, when the operator T has the null kernel and the norm $\|\cdot\|_*$ is defined by $\|x\|_* = \|Tx\|_Y$.

In general case representation (1.52) is incorrect, because replacing the spline smoothing problem on the space X by smoothing on the subspace $N(T)_*^\perp$ we obtain the different problem, which solution is not a solution to the original problem. It is easy to construct an example in 3-dimensional space X which shows this fact.

4. In the formulation of Theorem 4.3 the *weak* convergence of the subspaces E_k to X under constraint $E_k \subset E_{k+1}$ should be replaced by the *strong* one, because it can be easily shown that, if $E_k \subset E_{k+1}$, $k \in \mathbb{N}$, then

$$E_k \stackrel{w}{\to} X \iff E_k \to X \iff \bigcup_{k=1}^{\infty} E_k \text{ is dense in } X.$$

More precise results for the convergence of splines on the subspaces are given in [3].

5. In the proof of the lemma in Section 4.1.3 the final estimate for the norm of spline $\sigma^h_{\tau(h)}$ may be improved upon the following:

$$\|\sigma^h_{\tau(h)}\|_* \le (1 - C^2)^{-1/2} \|\varphi_*\|_*.$$

This follows from the estimate $\|M^h_{\tau(h)}\|_* \le (1 - C^2)^{-1}$ (cf. [3]) and the identity $\|\sigma^h_{\tau(h)}\|_*^2 = (M^h_{\tau(h)}\varphi_*, \varphi_*)_*$.

Similarly, we can obtain for splines in the thin layer (see Section 6.3.1) the estimate

$$\|\sigma^\Omega_h\|_{x(\Gamma)} \le \frac{C_2(\Omega)}{C_1(\Omega)} \times \|\varphi_*\|_{x(\Gamma)}$$

instead of (6.39).

References

[1] A.Yu. Bezhaev, V.A. Vasilenko, *Variational Spline Theory*, Bulletin of the Novosibirsk Computing Center, Series: Num. Anal., Special issue 3, NCC Publisher, Novosibirsk, 1993.

[2] A.I. Rozhenko, *Mixed spline approximation*, Bulletin of the Novosibirsk Computing Center, Series: Num. Anal., Issue 5, 1994, 67–86, NCC Publisher, Novosibirsk.

[3] A.I. Rozhenko, *Convergence of variational splines I*, Bulletin of the Novosibirsk Computing Center, Series: Num. Anal., Issue 6, 1994, 75–88, NCC Publisher, Novosibirsk.

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