

## Preface

The variational spline theory which originates from the well-known paper by J.C. Holliday (1957) is today a well developed field in the approximation theory. The general definition of splines in the Hilbert space, existence, uniqueness and characterization theorems were obtained about 20 years ago by M. Atteia, P.J. Laurent, P.M. Anselone, but in the recent years the important new results have been obtained in the abstract variational spline theory. These concern the convergence in the Hilbert spaces, general techniques for error estimation of abstract splines including multi-dimensional splines on scattered meshes, new kinds of characterization theorems based on the reproducing kernels and mappings, the theory and algorithms for the splines on subspaces (finite element method for complicated non-polynomial splines). New variational formulations arose for vector splines, rational splines, tensor and blending splines, traces of splines on the smooth manifold, discontinuous splines, etc. Optimal approximations of linear functionals and operators are also developed from these concepts.

These significant achievements are presented in this book in brief but complete form. Two monographs "Spline functions: theory, algorithms, programs" (1983) and "Spline functions and digital filters" (1984) and authors papers for recent 10 years have been used in this book. To a certain extent this text grew out of the course given to undergraduates at Novosibirsk University. The main language of this book is the functional analysis, but many practical examples are included. The book is intended for specialists in numerical analysis and also for students in pure and applied mathematics. We hope that this book will serve a powerful impulse in theoretical studies of splines, new applications and software developments.

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