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Monte Carlo simulation of multiple-scattered lidar returns (software description)*

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Abstract. The paper deals with Monte Carlo simulation of the lidar return signals. Descriptions of mathematical model, simulation algorithms, and mathematical software are presented as well as results of several numerical experiments for ground-based and space-borne lidars.

Keywords: Monte Carlo simulation, lidar return signals, ground-based and space-borne lidars, mathematical software.

Introduction

The lidar remote sensing is an effective tool to study optical and microphysical properties of scattering media. That is why solving the corresponding direct and inverse mathematical problems is of essential interest [1, 3, 7, 9, 12, 14, 16, 17]. This paper describes a free mathematical software, available on the author's web-page http://osmf.sscc.ru/~smp that can be used to investigate lidar return signals multiply scattered in a homogeneous layer (or a cylinder) for different media, signal characteristics, various monostatic and bistatic lidar schemes. The software is based on Monte Carlo method, which is one of the most promising approaches to simulate radiation transfer processes and lidar returns [1, 2, 8, 10–13]. A few numerical examples for ground-based and space-borne lidars are presented in this paper.

1. Statement of the problem

Assume that the following information is specified (Figure 1):

An optically isotropic scattering medium is concentrated in a horizontal plane-parallel layer $z \in [D-H, D]$, where $z = D$ is the upper boundary, $z = D - H$ is the lower boundary and H is the height of the layer. The optical properties are the same for the whole layer and they are described by an extinction coefficient σ, a single scattering albedo q, and a phase function $g(\mu)$, where μ is the cosine between directions of the beam of light before and after scattering.

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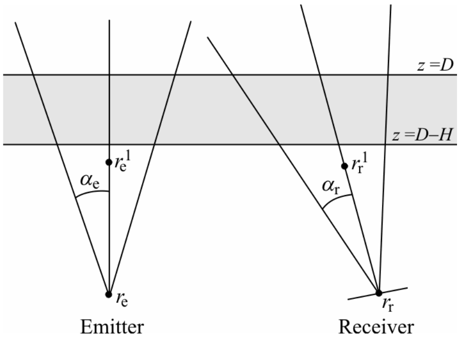

Figure 1. The lidar setup

The emitter of the lidar is placed at the point $r_e = (0, 0, 0)$. The beam of light propagates from the emitter in the direction $r_e^1 - r_e$, where r_e^1 is a point on the emitter semi-axis. Its energy is uniformly distributed with respect to the solid angle of the emitter field of view with a given semi-aperture α_e and with respect to the pulse-length l_e (which corresponds to the pulse-duration l_e/c in time, where $c = 0.3$ m/ns is the velocity of light). A laser pulse begins at the point $t = 0$ in time.

The receiver of the lidar with a semi-aperture α_r is situated at the point r_r. The direction of the receiver field of view is defined by a point r_r^1 fixed on its semi-axis. The receiver is a circle perpendicular to the vector $r_r^1 - r_r$ with radius R_r, its sensitivity being independent of the direction of the incoming light.

The objective is to compute the time dependence of a signal detected by the lidar receiver. In fact, the considered software calculates densities of the lidar return signal with respect to the time scale and with respect to the "photon path length" (which is equal to the time multiplied by the velocity of light). The second scale is particularly convenient for monostatic lidars when photon path length is equal to the doubled distance between the lidar and the emitted pulse of light. In addition to the densities, the absolute values of the energy returned to the receiver is computed for small intervals of time (and the photon path length) provided that the entire energy of the emitted pulse is equal to 1.

2. Monte Carlo algorithm

A numerical algorithm was developed based on a single local estimator and Monte Carlo simulation of photons trajectories to compute multiple-scattered lidar return signals. The trajectories of photons were simulated according to a well-known scheme [5, 11]:

Step 0. The initial point $r_0 = (x_0, y_0, z_0) = r_e$ of a photon, its weight $w_0 = 1$, and initial direction $\omega_0 = (a_0, b_0, c_0)$ in the emitter field of view are simulated, $\|\omega_0\|^2 = a_0^2 + b_0^2 + c_0^2 = 1$; $n = 0$.

Step 1. The photon free-path length l is simulated according to the probability density

$$p(l) = \sigma(r(l))e^{-\tau(l)}, \quad l > 0.$$

$$\tau(l) = \int_0^l \sigma(r(t))\, dt, \quad r(t) = r_n + t\omega_n,$$

where $\sigma(r)$ is the extinction coefficient at the point r and $\tau(l)$ is called an optical length or thickness of the interval $(r_n, r(l))$.

Step 2. Let us set $n := n + 1$ and calculate the coordinates x_n, y_n, z_n of the next collision point $r_n = (x_n, y_n, z_n)$:

$$x_n = x_{n-1} + a_{n-1}l, \quad y_n = y_{n-1} + b_{n-1}l, \quad z_n = z_{n-1} + c_{n-1}l.$$

Step 3. The scattering of a photon is simulated: a new direction of the photon ω_n is simulated according to the phase function $g(\mu)$, and the weight is recalculated by the formula $w_n = w_{n-1}q$. Then go to Step 1.

A trajectory terminates if a photon leaves a scattering medium or the 'intensity' w_n of the photon becomes negligible (less than a small value w^0).

A single local estimator is computed in the following way (see for detail [6, 10, 11]). The values

$$L = w_{n-1} q(r_n) e^{-\tau(r_n, r_{\mathrm{r}})} \frac{g(\mu)}{2\pi} \frac{\pi R^2 \chi}{|r_n - r_{\mathrm{r}}|^2} \tag{1}$$

are calculated for all photon trajectories and every collision point r_n in the receiver field of view. Here $\tau(r_n, r_{\mathrm{r}})$ is optical thickness of the interval (r_{r}, r_n), μ is a cosine of the angle between the vector $r_{\mathrm{r}} - r_n$ and the direction of the photon trajectory before scattering at the point r_n, and χ is a cosine of the angle between the vectors $r_{\mathrm{r}}^1 - r_{\mathrm{r}}$ and $r_n - r_{\mathrm{r}}$. The values L are added to a histogram of the lidar return signal according to the moments of time $t_n + |r_{\mathrm{r}} - r_n|/c$, where t_n is time of scattering at the point r_n.

Let us explain expression (1) of the local estimator. For every collision (if it is seen in the receiver field of view), a contribution to the lidar return signal is made proportional to the radiation scattered in the spatial angle that the receiver subtends at the collision point r_n. This contribution should be calculated as an integral over the spatial angle. Expression (1) is an approximation of this integral provided that (a) the spatial angle, which the receiver subtends at the collision point, is small enough, and (b) the area of the receiver projected on the plane perpendicular to the vector $r_{\mathrm{r}} - r_n$ is equal to $\pi R^2 \chi$.

Remarks. 1. The values L of the local estimator should be considerably smaller in comparison with the photons weight.

2. The lidar return signals corresponding to the single and double scattering of light can be computed without Monte Carlo simulation (see, e.g., [15]). But for the multiple-scattered radiation Monte Carlo method is indispensable.

3. Computational experiments

Figures 2–4 show the return signals computed by Monte Carlo method for a few lidar setups for single, double and higher-order scattering. It is assumed that the scattering medium in the layer has an extinction coefficient $\sigma = 0.02\ \mathrm{m}^{-1}$, a single scattering albedo q is equal to 1, and a phase function $g(\mu)$ corresponds to the cloud model C1 from [4] for the wavelength 0.6 µm. For the unit emitted energy, Figures 2–4 present values of the energy obtained by the receiver during short time intervals Δt.

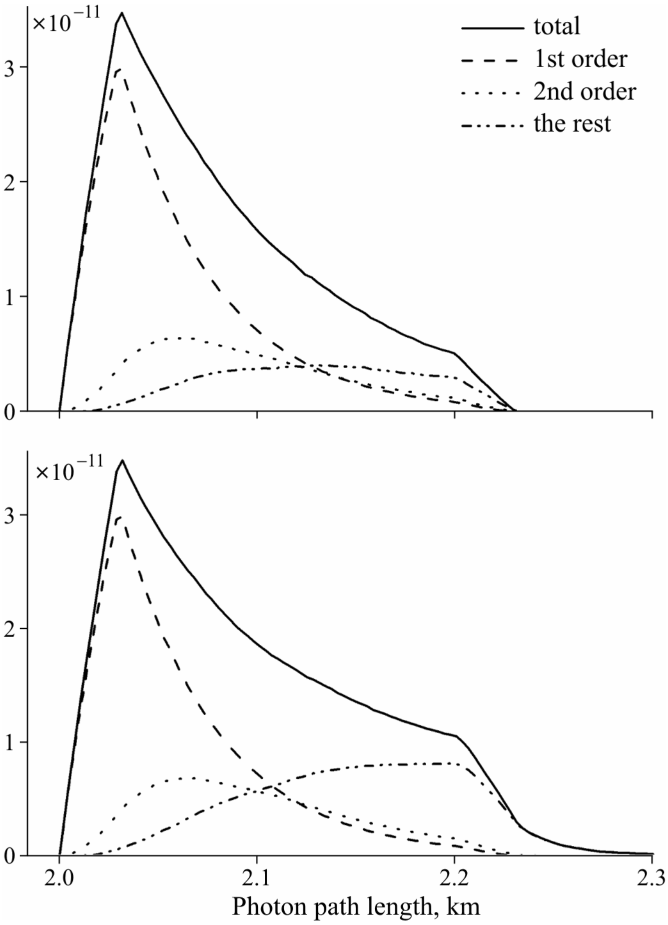

Figure 2. Simulated return signals for monostatic ground-based lidar with parameters: $H = 100$ m, $D = 1100$ m, $r_{\mathrm{e}} = r_{\mathrm{r}} = (0,0,0)$, $r^1_{\mathrm{e}} = r^1_{\mathrm{r}} = (0,0,1)$, $l_{\mathrm{e}} = 30$ m, $\alpha_{\mathrm{e}} = 5$ mrad, $c\Delta t = 3$ m, $\alpha_r = 10$ mrad (the upper figure) and $\alpha_r = 100$ mrad (the lower figure). The results were obtained by Monte Carlo simulation of 50 million photons trajectories

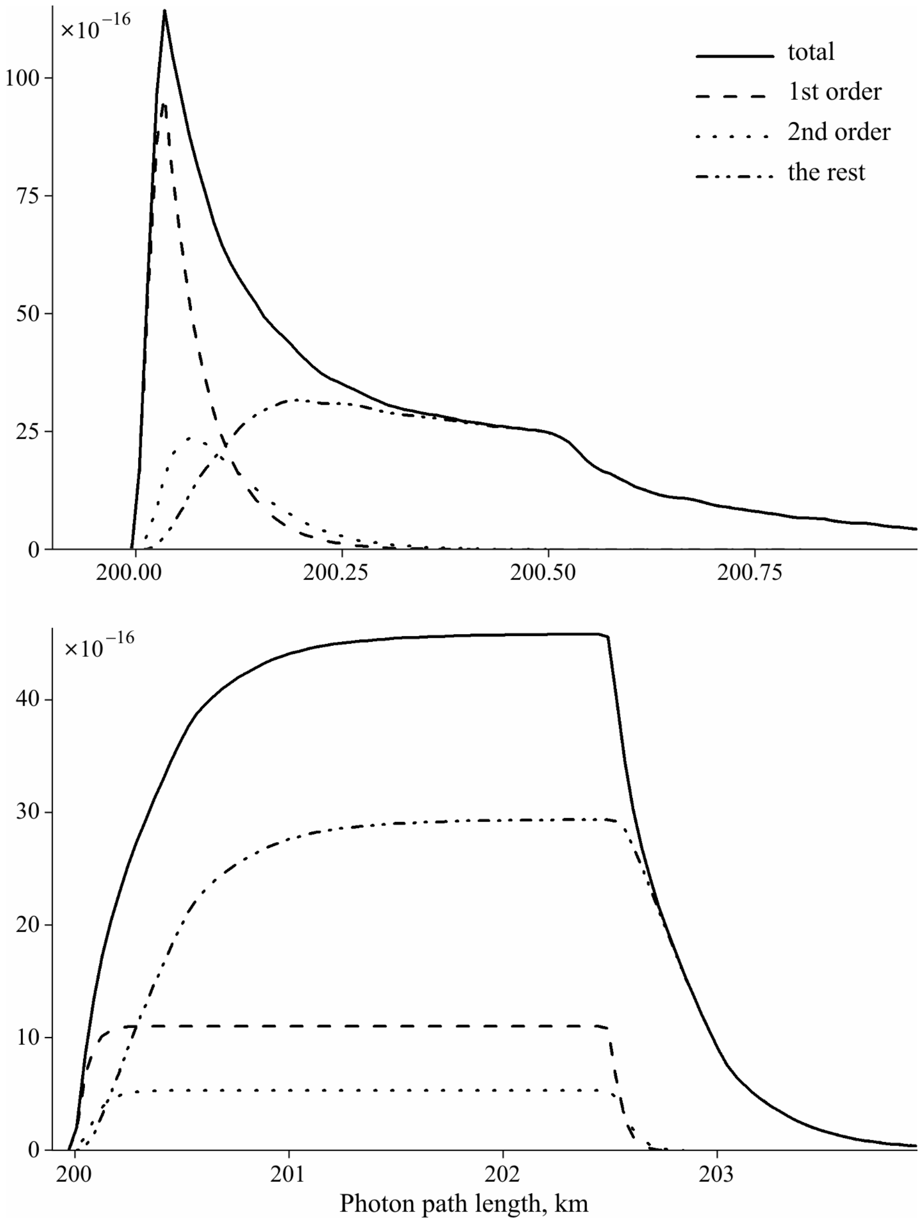

Figure 3. Simulated return signals for monostatic space-borne lidar with parameters: $H = 250$ m, $D = -100000$ m, $r_\mathrm{e} = r_\mathrm{r} = (0, 0, 0)$, $r^1_\mathrm{e} = r^1_\mathrm{r} = (0, 0, -1)$, $\alpha_\mathrm{e} = 5$ mrad, $\alpha_\mathrm{r} = 10$ mrad, $l_\mathrm{e} = 30$ m (the upper figure, $c\Delta t = 10$ m) and $l_\mathrm{e} = 2500$ m (the lower figure, $c\Delta t = 40$ m). The results were obtained by Monte Carlo simulation of 100 million photons trajectories

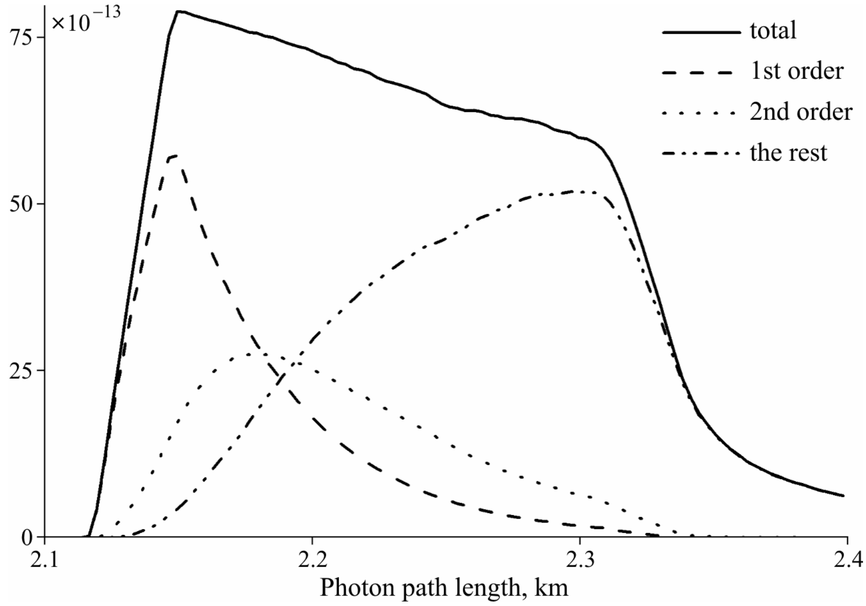

Figure 4. Simulated return signals for bistatic ground-based lidar with parameters: $H = 100$ m, $D = 1100$ m, $r_{\mathrm{e}} = (0,0,0)$, $r_{\mathrm{e}}^1 = (0,0,1)$, $r_{\mathrm{r}} = (500,0,0)$, $r_{\mathrm{r}}^1 = (500,0,1)$, $l_{\mathrm{e}} = 30$ m, $\alpha_{\mathrm{e}} = 5$ mrad, $c\Delta t = 3$ m, $\alpha_{\mathrm{r}} = 1500$ mrad. In this case, a component corresponding to the high-order scattering is responsible for the rectangular shape of the total signal. The results were obtained by Monte Carlo simulation of 100 million photons trajectories

The results of simulation illustrate how significant can be a contribution of multiple scattering to the return signals, especially, for bistatic and spaceborne lidars.

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References

[1] Bissonnette L.R., Bruscaglioni P., Ismaelli A., et al. Lidar Multiple Scattering from Clouds // Appl. Phys. B: Lasers Opt.—1995.—Vol. 60.—P. 355–362.

[2] Bruscaglioni P., Ismaelli A., Zaccanti G. Monte Carlo calculations of lidar returns: procedure and results // Appl. Phys. B: Lasers Opt.—1995.—Vol. 60.—P. 325–329.

[3] Bruscaglioni P., Flesia C., Ismaelli A., Sansoni P. Multiple scattering and lidar returns // Pure Appl. Opt: J. European Opt. Soc. A.—1998.—Vol. 7.—P. 1273–1287.

[4] Deirmendjian D. Electromagnetic Scattering on Spherical Polydispersions. — New York: American Elsevier, 1969.

[5] Ermakov S.M., Mikhailov G.A. Statistical Modelling. — Moscow: Nauka, 1982 (in Russian).

[6] Kalos M.H. On the estimation of flux at a point by Monte Carlo // Nuclear Science and Engineering. — 1963. — Vol. 16. — P. 111–117.

[7] Klett J.D. Stable analytical inversion solution for processing lidar returns // Appl. Opt. — 1981. — Vol. 20. — P. 211–220.

[8] Kunkel K.E., Weinman J.A. Monte Carlo analysis of multiply scattered lidar returns // J. Atmos. Sci. — 1976. — Vol. 33. — P. 1772–1781.

[9] Lu X., Jiang Yu., Zhang X., He Y. An algorithm to retrieve aerosol properties from analysis of multiple scattering influences on both ground-based and space-borne lidar returns // Optics Express. — Vol. 17, Iss. 11. — P. 8719–8728.

[10] Marchuk G.I., Mikhailov G.A., Nazaraliev M.A., et al. Monte Carlo Methods in Atmosperic Optics. — Springer, 1989.

[11] Mikhailov G.A. Some Problems of the Theory of Monte Carlo Methods. — Nauka: Novosibirsk, 1974 (In Russian).

[12] Oppel U.G., Wengenmayer M., Prigarin S.M. Monte Carlo simulations of polarized CCD lidar returns // J. Atmospheric and Oceanic Optics. — 2007. — Vol. 20, No. 12. — P. 1086–1091.

[13] Platt C.M.R. Remote sounding of high clouds. III: Monte Carlo calculations of multiple-scattered lidar returns // J. Atmos. Sci. — 1981. — Vol. 38. — P. 156–167.

[14] Ruppersberg G.H., Kerscher M., Noormohammadian M., et al. The influence of multiple scattering on lidar returns by cirrus clouds and an effective inversion algorithm for the extinction coefficient // Contributions to Atmospheric Physics. — 1997. — Vol. 70. — P. 93–105.

[15] Samokhvalov I.V. Double scattering approximation of lidar equation for inhomogeneous atmosphere // Opt. Lett. — 1979. — No. 5. — P. 12–14.

[16] Wang X., Boselli A., D'Avino L., Velotta R., et al. An algorithm to determine cirrus properties from analysis of multiple-scattering influence on lidar signals // Appl. Phys. B: Lasers Opt. — 2005. — Vol. 80. — P. 609–615.

[17] Winker D.M., Poole L.R. Monte Carlo calculations of cloud returns for ground-based and space-based lidars // Appl. Phys. B: Lasers Opt. — 1995. — Vol. 60. — P. 341–344.

Appendix. A brief description of the software

In order to simulate the multiple scattered lidar returns by Monte Carlo method, the mathematical software was developed for Microsoft Windows computers. The program was written in Pascal and compiled by Free Pascal Compiler (version 2.4) for computers with processors Pentium-2, Pentium-M, AMD. The executable file `lidar-1.exe` must be in the same directory with two input files `lidar-1.inp` and `phf_1.dat`. File `lidar-1.inp` includes input parameters for the program, and file `phf_1.dat` contains a table with the phase function of the scattering medium.

The results of computations are temporal return signals of the lidar (total and for different scattering orders). The square root of the variance is estimated for the total return signal. These estimates can be used to analyze the accuracy of Monte Carlo simulation.

After every batch of simulated photon trajectories, the results are recorded into the file `lidar-1.out` and several other additional files. The lidar temporal return signals are presented as absolute values of the energy detected by the receiver during small time steps and as densities with respect to the photon path length and time.

Let us mention some additional features of the program:

1) The return signals can be calculated not only for a layer with a scattering medium, but for a cylinder with a given radius as well. The restriction is that the emitter must be situated on the axis of the cylinder.

2) There is a possibility to continue Monte Carlo simulation after termination of the program (auxiliary files with `.tmp` extension are created during program running to realize this possibility). A random number generator used in the course of the program has a period exceeding 10^{22}. This is sufficient for the simulation of photons trajectories with trillions of scatterings.

3) The program can be started in the console or in the graphical mode (with additional visualization of current results).

4) If a receiver is situated in the scattering medium, then deviation of the local estimator can be large because of contributions from scatterings in the neighborhood of the receiver. The software enables one to 'introduce' a vacuum sphere around the receiver to diminish deviations in such cases.

5) Three different parts of the code are responsible for computations with the following characteristics of the emitted pulse: (a) $\alpha_{\mathrm{e}} = 0$, $l_{\mathrm{e}} = 0$, (b) $\alpha_{\mathrm{e}} > 0$, $l_{\mathrm{e}} = 0$, (c) $\alpha_{\mathrm{e}} \geq 0$, $l_{\mathrm{e}} > 0$.

6) The current version of the software has the following restrictions: the fixed distribution of the emitted laser pulse in time and solid angle, simplified characteristics of the receiver, an optically isotropic scattering medium in a homogeneous single layer (cylinder), polarization not being taken into account.

7) A simple protection technique was realized in program LIDAR-1 to prevent unauthorized modification of the executable code.

8) For some cases, Monte Carlo simulation is a time consuming process, and the estimate of the return signal is noisy. A special technique of dependent splitting of trajectories with respect to the first scattering was implemented, and, hopefully, sometimes it can improve the results of simulation.

The software is still under development. Any feedback is highly appreciated. The program can be downloaded from http://osmf.sscc.ru/~smp/LIDAR-1.WEB.ZIP.