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Inverse problems of estimating radioactive impurities of the Enisei river basin*

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Abstract. When handling and interpreting results of experimental studying radioactive impurities of rivers using methods of formulation of both direct and inverse problems as related to impurities transport, there can arise essential difficulties associated with inadequacy of mathematical models used to available observations data. With direct modeling of radionuclides in a river bed, it is necessary to set a large number of hydrological parameters, characteristics of nuclide distribution in the systems "water-suspension" and "water-bottom sedimentation", etc. Eventually, all the above-said can essentially narrow down the areas of application of these models in spite of their rather a universal nature [1]. When using statements of inverse problems, it is undesirable to describe in great detail the migration processes of radionuclides, because this can cause serious difficulties of their substantiation and numerical realization.

1. Statement of inverse problem

Let the concentration of a substance run-off released into the river vary due to the impact of the following factors: scattering, advection, and decay. The propagation process will be described by the following differential equation of second order [2]

$$\frac{\partial}{\partial x}\left(Qc - EA\frac{\partial c}{\partial x}\right) + \lambda Ac - f(x) = 0, \tag{1}$$

where x is a longitudinal coordinate along the river bed; Q is water flux; E is a longitudinal dispersion factor; A is the area of the cross-section of the flow; λ is a constant of the radioactive decay rate; $f(x)$ is a source, characterizing the arrival of impurity into the river, with supplementary conditions

$$c|_{x=x_0} = 0, \qquad c \to 0 \quad \text{when} \quad x \to \infty. \tag{2}$$

Let us introduce the notation

$$\phi = Ac. \tag{3}$$

Then, with allowance for $Q = uA$, from (1) we obtain

$$E\frac{\partial^2\phi}{\partial x^2} - \left(u + \frac{EA'}{A}\right)\frac{\partial\phi}{\partial x} - \left[\lambda + u' + E\left(\frac{A'}{A}\right)'\right]\phi + f(x) = 0. \tag{4}$$

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Here u is mean velocity of the radionuclide transport. In case when $A(x)$ weakly depends on x ($A'(x) \approx 0$), u and E are constants, from (4) follows the following problem:

$$\frac{\partial^2 \phi}{\partial x^2} + a\frac{\partial \phi}{\partial x} + b\phi = g(x), \tag{5}$$

$$\phi|_{x=x_0} = 0, \qquad \phi \to 0 \quad \text{when} \quad x \to \infty, \tag{6}$$

where

$$a = -\frac{u}{E}, \quad b = -\frac{\lambda}{E}, \quad g(x) = -\frac{f(x)}{E}.$$

Since $a^2 > 4b$, the solution to problem (5), (6) can be represented as

$$\phi(x, \vec{\theta}) = \theta_1 \int_{x_0}^{x} g(\xi) e^{\theta_2(x-\xi)} d\xi, \tag{7}$$

where $\theta_1 = 2/(E\sqrt{a^2 - 4b})$, $\theta_2 = (-a - \sqrt{a^2 - 4b})/2$. From analysis of dependence (7) it follows that in order that the vector of the parameters $\vec{\theta}$ and $\phi(x, \vec{\theta})$ be estimated, it is necessary to take measurements of the impurity concentration from not less than two section lines of the river. If the function $f(x)$ describes a point impurity source of exposure M, i.e., $f(x) = M\delta(x)$, then from (7) obtain

$$\phi(x, \vec{\theta}) = M\theta_1 \cdot e^{\theta_2 x}. \tag{8}$$

With allowance for (2), (8), we arrive at the following regression dependence for the impurity concentration

$$c(x, \vec{\theta}) = \frac{\theta_1'}{A(x)} e^{\theta_2 x}, \tag{9}$$

where $\theta_1' = M\theta_1$. Estimations of the parameters θ_1 and θ_2 can be obtained with the use of measurements data of the radionuclide concentration on various section lines, for example, by the least square method [3].

Remark. If a term with the longitudinal dispersion E is neglected in equation (1), the impurity concentration can be described by the relation

$$c(x, \theta_1, \theta_2) = \frac{\theta_1}{Q(x)} \exp(-\theta_2 \cdot x), \tag{10}$$

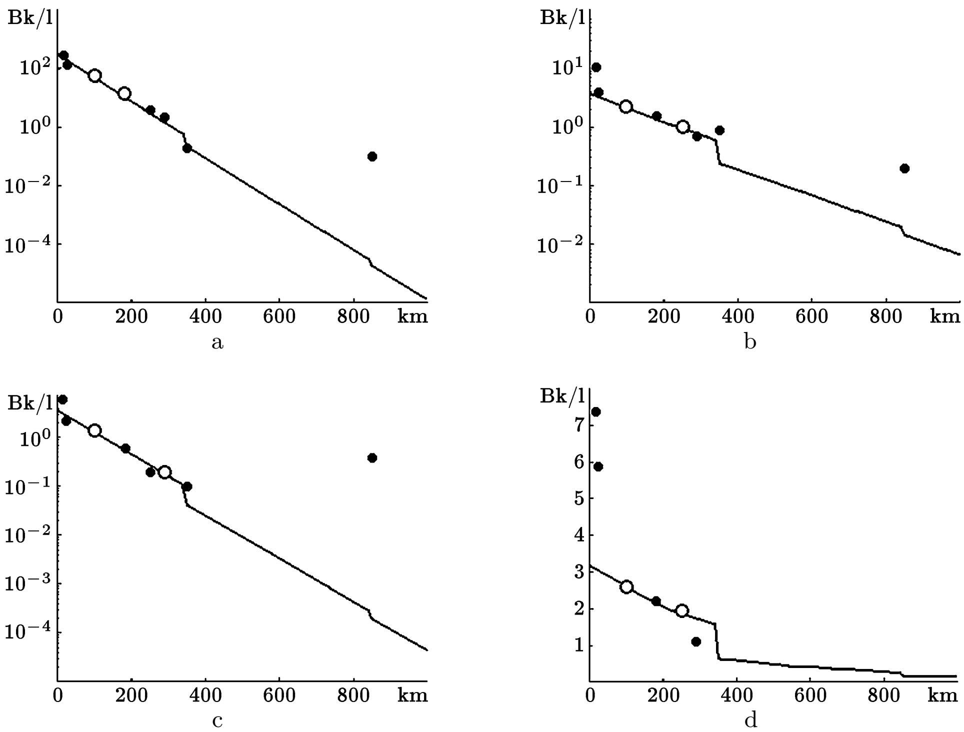
where θ_1 is a value, proportional to the discharge exposure,

$$\theta_2 = \frac{0.693}{T_{0.5} u}, \tag{11}$$

$T_{0.5}$ is a half-decay time, u is the mean velocity of nuclide migration, c is concentration (Bk/l), $Q(x)$ is water flux.

2. Reconstruction of impurities of the river Enisei by observation data

In order that the radiation circumstances of the river Enisei from the city of Krasnouarsk to the village Bor be studied, field experiments of contamination of water, fish, bottom sedimentation, flood plain were carried out in the August of 1991 [4]. Altogether, there were investigated more than 20 section lines. Based on statements of inverse problems associated with impurities transport, the obtained experimental data on water contamination are analyzed. As basis relations, regression dependencies (9)–(11) are used. The figure represents the results of reconstruction of contamination by two section lines, which are marked with light circles. The observations data for other section lines are marked with dark circles and can be used for monitoring the accuracy of reconstruction. Analysis of the figure implies that within certain intervals the agreement between calculations and observations is fairly satisfactory. The best agreement is observed for short-lived radionuclides ^{24}Na, ^{76}As, ^{239}Np. The discrepancy between calculations and observations (less than 100 km from the site of discharge) is due to insufficient uniformity of nuclide distribution over the river section.



Radionuclide concentration in the river Enisei with allowance for suspension: (a) ^{24}Na, (b) ^{239}Np, (c) ^{76}As, (d) ^{32}P

Table 1. Estimations of regression parameters (10)

Radionuclide	Estimation of		Half-decay time $T_{0.5}$, hours
	θ_1	θ_2	
^{24}Na	1070	1.8	15
^{76}As	12.3	1.0	26.3
^{239}Np	11.7	0.5	56
^{51}Cr	10.0	0.19	344
^{32}P	37.5	0.41	666

Table 2. Mean velocities of radionuclide migration

Radionuclide	Migration rate, km/h
^{24}Na	2.57
^{76}As	2.63
^{239}Np	2.48
^{51}Cr	0.25
^{32}P	1.06

The results of estimating parameters are presented in Table 1. Table 2 with the help of the estimations θ_2 and relation (11) presents mean velocities of nuclide migration downstream the river. For the short-lived nuclides, these velocities practically coincide. For ^{32}P, ^{51}Cr the migration velocities appeared to be essentially lower and require a supplementary study.

3. Optimal planning of observations

For the definiteness sake, let us restrict ourselves to working out the plans maximizing the determinant of the informational matrix [3]. Using the definition of the informational matrix and the necessary extremum condition, the relation between points of the optimal plan of observations and unknown parameters of regression (10) can be presented in the analytical form:

$$(x_2 - x_1)\left[\theta_2 + \frac{Q'(x_2)}{Q(x_2)}\right] = 1. \tag{12}$$

With fixed x_1, the optimal localization of the section line x_2 is determined from solution to equation (12). For the case $Q(x) = \text{const}$ from (12) follows

$$x_2 = x_1 + 1/\theta_2. \tag{13}$$

Relation (13) shows that independent of selection of the section line x_1, a distance between the section lines x_1 and x_2 should be constant. According to (11), the value of this distance is proportional to the half-decay time of a concrete radionuclide. Using the estimations of the parameters θ_2, given in Table 2, and the iterative procedure of sequential planning for regressions (9), (10), the locally optimal plans of measurements were numerically simulated. The admissible domain for taking measurements was set according to supposition about uniform distribution of radionuclides over the river section. Such a supposition is valid at distances about 100 km from the discharge site [4], thus providing quite an adequate description of transport processes using relations (9), (10).

The results of simulation are presented in Table 3. Analysis of Table 3 implies that in the case of ^{24}Na, ^{76}As, ^{239}Np, the locally optimal plans practically coincide both for $Q(x) = \text{const}$, and for a real water flux in the river. In the case of ^{51}Cr and ^{32}P, distinctions between localizations of the points x_2 appear to be clear-cut due to the issue of the river Angara, thus bringing about a sharp change in $Q(x)$.

Table 3. Locally optimal plans of measuring radionuclide concentration

Radionuclide	$R(x)$, real		$R(x) = \text{const}$	
	x_1	x_2	x_1	x_2
^{24}Na	100	155	100	155
^{76}As	100	176	100	180
^{239}Np	100	216	100	220
^{51}Cr	100	250	100	350
^{32}P	100	250	100	600

4. Conclusion

The models of radioactive water contamination, based on statements of inverse problems of impurities transport, make it possible to adequately reconstruct a continuous picture of radionuclide propagation by observations on a limited number of section lines. The estimations of mean velocity of the downstream propagation of the short-lived nuclides ^{24}Na, ^{76}As, ^{239}Np. The locally optimal schemes of the section lines distribution have been numerically constructed for measuring concentrations of various radionuclides in water.

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