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Numerical research into channel currents with a flat plain model*

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1. Introduction

Mathematical modeling is a tool of the prime importance for the run-off formation study. It gains in importance under condition of a sparse and out-of date monitoring network typical of majority of the Siberia regions. In analyzing the spatial-temporal laws of the run-off formation in large river basins, the main problems are associated with the absence of a basic hydrological model capable of treating real hydrological and hydro-chemical observation data.

A method of solution for variable-depth flows in arbitrarily-shaped domains are currently underestimated. The water level rise and fall can result in a change of the shape of an area due the islands submerging, sandbanks emerging, inundation of floodplains, etc. This requires the formulation of the edge problem with unknown boundaries, tracing through all wetted perimeters, and foreseeing the formation of new internal boundaries in the form of islands due to the shallow water places drying up. Conventional methods of solving such problems (e.g., the fictitious domain method) encounter severe algorithmic difficulties associated with problem degeneracy under condition of water layer disappearance, with development of multilogic systems, and with excessive intellectualization of a program.

Comprehensive methods of computational mathematics allow us to avoid these difficulties through the use of monotone numerical schemes. The monotonicity property itself provides the non-negativity of calculated values such as a water layer thickness h, the Celsius temperature, admixture concentration. The application of monotone scheme, for example, h, ensures the fulfilment of the relation $h \geq 0$ within the whole definition domain including shallow waters and dry places where $h = 0$. Hence, such an approach does not require building the algorithm of non-calculated areas formation, but allows us to assume $h = 0$ for dry places and to integrate the equations through the whole domain. The external boundary in this case can be located along the watershed line or assigned according to topography peculiarities.

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2. Basic equations

The below-formulated hydrodynamic model formulated below is designed for calculation of a flow field, run-offs, levels, turbulence characteristics, and a free water surface shape in natural reservoirs of an arbitrary configuration or in their parts.

The majority of lakes and water storage reservoirs in Western Siberia are characterized as shallow (a mean depth of 2–4 m) and flat-bottomed, with shallow-incised beds. Thermal and density stratification of such reservoirs is not significant, and currents are generated mainly due to the wind forcing and bottom slope. For such conditions, we can assume that processes are hydrostatic and proceed to a flat horizontal problem [1, 2].

Let us introduce the Cartesian coordinates with x-, y-, and z-axes, where z-axis is directed upward. Assign the sea level of reservoir (including channel beds) by the equation $z = \delta(x, y)$. The initial equations are considered in the area, whose shape can change with time, e.g. as a result of a water level rise or a storage decrease.

If a current is assumed to be turbulent, then the equations of horizontal (plain) movement of water are derived from the Reynolds equations by averaging along the vertical axis. Consider the velocities averaged along z-axis $\bar{u} = \frac{1}{h}\int_{\delta}^{h} u\,dz$, $\bar{v} = \frac{1}{h}\int_{\delta}^{h} v\,dz$, where u, v are components of the horizontal velocity vector along the axis x and y; h is the flow depth. After elimination of the bar over variables, the system of plain current equations can be written down as follows:

$$\begin{aligned}
&\frac{\partial(hu)}{\partial t} + \frac{\partial(huu)}{\partial x} + \frac{\partial(huv)}{\partial y} \\
&\qquad = -gh\frac{\partial(h+\delta)}{\partial x} + \frac{\partial}{\partial x}hK_x\frac{\partial u}{\partial x} + \frac{\partial}{\partial y}hK_y\frac{\partial u}{\partial y} + \tau_x - r|\vec{u}|u, \\
&\frac{\partial(hv)}{\partial t} + \frac{\partial(huv)}{\partial x} + \frac{\partial(hvv)}{\partial y} \\
&\qquad = -gh\frac{\partial(h+\delta)}{\partial y} + \frac{\partial}{\partial x}hK_x\frac{\partial v}{\partial x} + \frac{\partial}{\partial y}hK_y\frac{\partial v}{\partial y} + \tau_y - r|\vec{u}|v, \\
&\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} + \frac{\partial(vh)}{\partial y} = R_a - I,
\end{aligned} \tag{1}$$

where τ_x, τ_y are wind stresses, r is a bottom friction coefficient, $|\vec{u}|$ is a current velocity module, K_x, K_y are turbulence coefficients, R_a is the source intensity (precipitation), I is infiltration and evaporation.

Setting up the boundary conditions is done according to morphological characteristics of a reservoir and by the water exchange type. To describe a closed-loop reservoir the conditions of non-leakage and friction against the side walls of the channel bed are set for the lateral boundaries

$$u_n = 0, \quad K\frac{\partial u_s}{\partial n} = -c_u|\vec{u}|u_s \quad \text{for } (x, y) \in \Gamma, \tag{2}$$

where Γ is a domain boundary, n is the external normal to the boundary, u_n, u_s are normal and tangential components of the horizontal velocity vector, c_u is a resistance coefficient along the coastline. If there are open (liquid) boundaries, the discharges $Q^{(x)}$, $Q^{(y)}$ for assigned sections must be known:

$$hu = Q^{(x)}, \quad hv = Q^{(y)} \quad \text{for } (x, y) \in \Gamma, \tag{3}$$

or turbulent water flows must be set for the output section lines, e.g. in the form

$$hK\frac{\partial u}{\partial n} = hK\frac{\partial v}{\partial n} = 0 \quad \text{for}(x, y) \in \Gamma. \tag{4}$$

Modeling of the turbulent exchange is performed on the basis of a two-parametric system of equations of a semi-empirical theory of turbulence [3]. Equations for the density of the turbulence kinetic energy (TKE) e and for the TKE dissipation rate ε averaged along z-axis have the form

$$\begin{aligned}
&\frac{\partial(he)}{\partial t} + \frac{\partial(hue)}{\partial x} + \frac{\partial(hve)}{\partial y} \\
&\qquad = \alpha_e\left(\frac{\partial}{\partial x}hK\frac{\partial e}{\partial x} + \frac{\partial}{\partial y}hK\frac{\partial e}{\partial y}\right) + hKJ_{xy} - h\varepsilon + P_e, \\
&\frac{\partial(h\varepsilon)}{\partial t} + \frac{\partial(hu\varepsilon)}{\partial x} + \frac{\partial(hv\varepsilon)}{\partial y} \\
&\qquad = \alpha_\varepsilon\left(\frac{K\varepsilon}{h} + \frac{\partial}{\partial x}hK\frac{\partial \varepsilon}{\partial x} + \frac{\partial}{\partial y}hK\frac{\partial \varepsilon}{\partial y}\right) + c_2 h\frac{\varepsilon}{e}KJ_{xy} - c_3 h\frac{\varepsilon^2}{e}, \\
&K = c_\mu\frac{e^2}{\varepsilon},
\end{aligned} \tag{5}$$

where P_e is a near-surface flow of the TKE induced by a wind wave, α_e, c_μ, c_2, c_3 are empirical constants [4], $J_{xy} = u_x^2 + u_y^2 + \bar{u}_z^2 + v_x^2 + v_y^2 + \bar{v}_z^2 - S_0$, S_0 is a parameter reflecting the water layer average stratification, $\bar{u}_z$, $\bar{v}_z$ parametrically describe the velocity gradients along z-axis. The last relation in (5) expresses the Kolmogorov closure hypothesis.

The system of equations of a subgrid-scale closure for plain currents was derived by means of vertical averaging of equations of spatial turbulence model [5]. Equations in the presented form describe, in particular, the known non-trivial solutions corresponding to a horizontally homogenous flow with a vertical shear of velocity (the wall-adjacent layer of constant flows). This circumstance permits the use of equations of $e\varepsilon$-model for the calculation of the horizontal turbulent exchange.

3. Methods of solution

The boundary approximation of the initial domain is fulfilled by projecting it onto a regular-cell grid space. The points having missed the domain are excluded from the numerical integration. The boundary contours represent a piecewise-linear surface formed by the finite elements of tangential hyperplanes. There are no severe restrictions for the boundary topology, and the domain multiconnection is assumed. The technology of digital reconstruction of boundaries is automated.

Methods of solving equations are based on discretization of initial systems in the grid space. Non-uniform rectangular grids are used with points spaced apart to the sides of an elementary simulation box. "The loose" grids allow building the conservative difference schemes, and applied implicit methods ensure the method stability in the coarse of the long-term integration [6].

The spatial approximation of differential operators is based on the current concepts of monotone or TVD-schemes (Total Variation Diminishing) [7].

4. Numerical experiments

The possibilities of model can be illustrated by calculation of a current within 3-km length of the Ob river in the region of Barnaul city water intake. Figure 1 shows the reservoir morphometry. The river width is 600–700 m, a maximum depth is 11 m. The interest attracted to this study lies in progressing the bottom load transport, which impedes an exchange in a submersible channel of the water intake. The problem is solved in a channel with rigid walls and a rough bottom. The given experiment is restricted by analysis of the steady-state conditions with an input section line discharge of 1300 m^3/s. In this case, the current function ψ can be introduced (isolines are shown in Figure 1). The flow is directed along the deep-water part of the channel to the north, and the current hugs the right bank. A zone of low velocities marked with closed lines of the current is formed after the channel bend. This zone formation is similar to that of detached flows in the hydrodynamic flow round an inverse bench.

The back-water zones with rotor currents are also formed in the river dead end (back-water in the upper part of Figure 1) and in the middle part of the domain. Velocity and discharge values are small in these zones, which is shown in a discharge diagram built for $x = 1200$ m (see Figure 1). In this part of the channel a maximum current velocity is approximately 50 cm/s, but it increases with approaching the output section line due to the channel narrowing and the wetted cross-section reduction. The friction-induced water level fall is 28 cm.

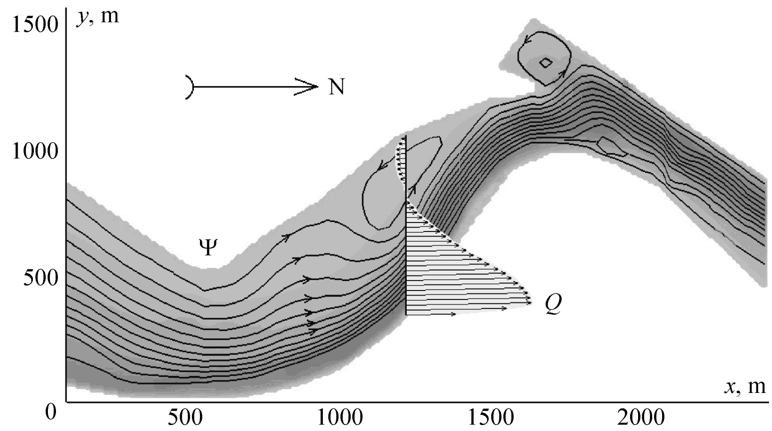

Figure 1. The current function isolines and the structure of a cross-section discharge (the central part) of the Ob river stretch near Barnaul city

The second example is a current in Novosibirsk water storage reservoir, for which necessary data are available. Processes in the submerged river valleys are characterized by a slow water exchange. Parameters of the water exchange in reservoirs strongly influence all the internal processes—channel, chemical, biological, etc. In this connection, a detailed current pattern in the limnetic part of the reservoir is critical, since different types of a current can be observed—from rapid deep-stream currents to still-water and back-water ones.

A digital bathymetric map of the reservoir for the area of 40×160 km^2 has 300-m resolution and contains about 70,000 points. Analysis of morphometric characteristics shows that there are marked channel and limnetic parts of the current downstream up to the section line of a hydroelectric power plant. In the valley part of the channel, in particular, near Zavyalovo village, there are many islands and sand-banks.

An attempt of the direct boundary approximation in a complex multi-connection domain runs across some severe difficulties during realization of boundary-value problem. For example, one boundary point near an island' prominence can be connected with two or more calculated near-boundary points. This results in the disturbance equation balance and boundary conditions and makes the boundary-valued problem overestimated, i.e., incorrect. So, the technique of one formulation of a problem without boundary conditions is used in this case. The calculation method allows one to solve such problems on the basis of the apparatus of non-oscillating interpolants. The model in such a presentation provides a uniform calculation of dry plots wetted by the water, a description of submerging islands, flood-lands, etc. without special isolation of dry or submerged areas and organization of curvilinear boundaries of a variable configuration.

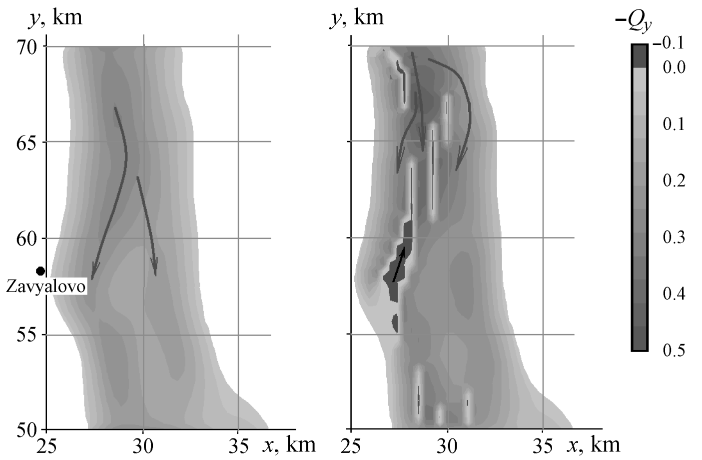

Figure 2. Spatial distribution of the discharge Q_y (m^2/s) for the channel length of Novosibirsk storage reservoir near Zavyalovo village in no-wind conditions (left) and with north-east wind (right)

Near to Zavyalovo village, there is Novopichugovskoye sand and pebble deposit currently developed with a suction dredger. The open mining can result in the channel deformation due entering suspended solids into the flow and their further transportation and deposition. So, the study of the current velocity mode and revealing possible ways of the admixture transport in this region is interesting.

The channel width in this part of the reservoir is 5–7 km. Figure 2 (left) presents the structure of y-component of the discharge vector $Q_y = hv$ at the channel stretch of 20 km of length in the vicinity of Zavyalovo village. At the entrance range of the channel, the full discharge was assigned as $Q = 1300$ m^3/s, which corresponds to a fall of low-water level. The calculated field of velocities obtained in the experiment was characterized by relatively small values, i.e., below 7 cm/s. At such velocities, the impact of surface wind stresses becomes significant. Thus, with a south-west wind (along the stream) of a force of 5 m/s, the flow velocity twofold increases which is consistent with observation data.

A moderate wind of 7 m/s speed of the opposite direction blowing upstream is responsible for a single jets' production — wind belts oriented upstream. Such jets are shown in Figure 2 (right). They are essentially nonstationary, i.e., they occur, migrate, dissipate, and result from interaction of two antagonistic factors — a channel flow and a contrary wind. Wind belts can be interpreted as coherent structures, i.e., eddy stochastic formations within the turbulence field, which keep stability for a definite time. It should be noted, however, that the phenomenon described above can be observable at a long-term (several hours) wind load.

5. Conclusion

The two-dimensional numerical model of plain currents is designed for discharge calculation and water levels in water streams and water basins with complex geometry. The numerical experiments show that the spatial structure of channel currents can be essentially inhomogeneous and include rotations and backwaters, as well as jets directed the other way to main stream.

The model can be applied as a tool for the description of the hydrological processes in the scale of local river basins, river channels with islands and floodplains of complex configuration, and for water management decisions.

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