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# Propagation of a gravity current in the atmosphere over a valley*

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**Abstract.** A 2D version of a 3D nonhydrostasic meteorological model is used to simulate the effects of atmospheric front propagation over a valley. The front surface is described in the model by an equation for advection of a scalar substance, which is solved by a third-order semi-Lagrangian procedure. A leap-frog type scheme is used for time discretization. Special-purpose operators of space discretization are used to provide conservation of momentum and scalars. The results of 2D model simulations show reasonable behavior of cold front propagation over a valley as compared to the available measurement and simulation data.

## 1. Introduction

The propagation of an atmospheric front over steep terrain is a phenomenon of great practical importance in meteorology[1–4]. This is also a subject matter of interest for numerical modelers, since atmospheric fronts can be considered as surfaces of discontinuity in the atmosphere. To simulate the deformation of these surfaces by spacial obstacles like mountains and valleys with good accuracy, efficient numerical methods are needed. The literature on theoretical studies of atmospheric fronts is not extensive (e.g. [5]). Two distinct approaches can be recognized in the numerical simulation of front propagation. In one approach, the front to be calculated is considered as a gravity current driven by a cold air source [6]. In the other, the front surface is considered as a passive scalar, a tracer to distinguish between warm and cold air masses [7].

In the present paper, a preliminary investigation is carried out to simulate cold front propagation over a steep valley in two dimensions. For this, a 2D version of a 3D nonhydrostasic meteorological model is used. The model is based on spacial discretizations that conserve some important quantities of the phenomena under study like momentum and scalars. Also, an efficient procedure is used to calculate the advection of scalars. The purpose of the present paper is to test these schemes in the case of cold front propagation over an idealized valley.

In Section 2, the basic model equations are formulated. In Section 3, these equations are discretized and some limitations on topography steepness

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necessary for numerical stability are given. In Section 4, the results of model simulations for orographic retardation of an idealized cold front by a valley are discussed.

## 2. The model

We consider here a small-scale nonhydrostatic model developed for simulations mainly of meso- and microscale flows (see, e.g., [8]). In three-dimensional statement, the basic equations of motion, heat, moisture, and continuity in a terrain-following coordinate system are as follows:

$$\frac{dU}{dt} + \frac{\partial P}{\partial x} + \frac{\partial (G^{13} P)}{\partial \eta} = f_1(V - V_g) - f_2 W + R_u,$$

$$\frac{dV}{dt} + \frac{\partial P}{\partial y} + \frac{\partial (G^{23} P)}{\partial \eta} = -f_1(U - U_g) + R_v,$$

$$\frac{dW}{dt} + \frac{1}{G^{1/2}} \frac{\partial P}{\partial \eta} + \frac{gP}{C_s^2} = f_2 U + g \frac{G^{1/2} \bar{\rho} \theta'}{\bar{\theta}} + R_w$$

$$\frac{d\theta}{dt} = R_\theta, \qquad \frac{ds}{dt} = R_s,$$

$$\frac{1}{C_s^2} \frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial}{\partial \eta} \Big( G^{13} U + G^{23} V + \frac{1}{G^{1/2}} W \Big) = \frac{\partial}{\partial t} \Big( \frac{G^{1/2} \bar{\rho} \theta'}{\bar{\theta}} \Big),$$

$U = \bar{\rho} G^{1/2} u$, $V = \bar{\rho} G^{1/2} v$, $W = \bar{\rho} G^{1/2} w$, $P = G^{1/2} p'$, where $p'$, $\theta'$ are deviations from a basic state pressure $\bar{p}$ and potential temperature $\bar{\theta}$, $s$ is the specific humidity, $C_s$ is the sound wave speed, $u_g$, $v_g$ are the components of a geostrophic wind representing the synoptic part of the pressure, $\eta$ is a terrain-following coordinate transformation:

$$\eta = \frac{H(z - z_s)}{(H - z_s)},$$

$z_s$ is the surface height, $H$ is the height of the top of the model domain. Here $H = \text{const}$;

$$G^{1/2} = 1 - \frac{z_s}{H}, \quad G^{13} = \frac{1}{G^{1/2}} \Big( \frac{\eta}{H} - 1 \Big) \frac{\partial z_s}{\partial x}, \quad G^{23} = \frac{1}{G^{1/2}} \Big( \frac{\eta}{H} - 1 \Big) \frac{\partial z_s}{\partial y}.$$

For an arbitrary function $\varphi$

$$\frac{d\varphi}{dt} = \frac{\partial}{\partial t} + \frac{\partial u\varphi}{\partial x} + \frac{\partial v\varphi}{\partial y} + \frac{\partial \omega\varphi}{\partial \eta},$$

where

$$\omega = \frac{1}{G^{1/2}} W + G^{13} u + G^{23} v.$$

The terms $R_u$, $R_v$, $R_\omega$, $R_\theta$, $R_s$ refer to subgrid-scale processes. As a turbulence parameterization, we use a simple scheme:

$$K_m = \begin{cases} l^2\sqrt{\dfrac{1}{2}D^2(1-\mathrm{Ri})}, & \mathrm{Ri} < 1, \\ 0, & \mathrm{Ri} \geq 1, \end{cases}$$

$$\mathrm{Ri} = \frac{g(d\ln\theta/dz)}{D^2/2}, \qquad D = \nabla\underline{u} + \underline{u}\nabla.$$

## 3. Discretization

We approximate the advective terms in the model described above by the following difference operators:

$$\begin{aligned}
\delta_d\varphi &= \frac{\varphi(d+\Delta d/2) - \varphi(d-\Delta d/2)}{\Delta d}, \\
\varphi^d &= \frac{\varphi(d+\Delta d/2) + \varphi(d-\Delta d/2)}{2}, \\
\mathrm{ADVX} &= \delta_x(u^x(\rho^x u)^x) + \delta_y(v^x(\rho^x u)^y) + \delta_z(\omega^x(\rho^x u)^z), \\
\mathrm{ADVY} &= \delta_x(u^y(\rho^y v)^x) + \delta_y(v^y(\rho^y v)^y) + \delta_z(\omega^y(\rho^y v)^z), \\
\mathrm{ADVZ} &= \delta_x(u^z(\rho^z w)^x) + \delta_y(v^x(v(\rho^z w)^y) + \delta_z(\omega^z(\rho^z w)^z), \\
\mathrm{ADVT} &= \delta_x(u(\rho\theta)^x) + \delta_y(v(\rho\theta)^y) + \delta_z(\omega(\rho\theta)^z)
\end{aligned}$$

In order to carry out a stability analysis of the von Neumann type, the basic equations system is linearized around a constant basic state wind velocity vector $(\bar{U}, \bar{V})$. Then the original equations are reduced to the following ones:

$$\begin{aligned}
&\frac{\partial U}{\partial t} + \frac{\partial P}{\partial x} = -\bar{U}\frac{\partial U}{\partial x} - \bar{V}\frac{\partial U}{\partial y} - \Delta G\frac{\partial P}{\partial \eta}, \\
&\frac{\partial V}{\partial t} + \frac{\partial P}{\partial y} = -\bar{U}\frac{\partial V}{\partial x} - \bar{V}\frac{\partial V}{\partial y} - \Delta G\frac{\partial P}{\partial \eta}, \\
&\frac{\partial W}{\partial t} + \frac{\partial P}{\partial \eta} = N\theta'' - \Delta H\frac{\partial P}{\partial \eta} - \bar{U}\frac{\partial W}{\partial x} - \bar{V}\frac{\partial W}{\partial y}, \\
&\frac{\partial \theta''}{\partial t} = NW - \bar{U}\frac{\partial \theta''}{\partial x} - \bar{V}\frac{\partial \theta''}{\partial y}, \\
&\frac{1}{C_s^2}\frac{\partial P}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial \eta} = -\Delta H\frac{\partial W}{\partial \eta} - \Delta G\frac{\partial U}{\partial \eta} - \Delta G\frac{\partial V}{\partial \eta}.
\end{aligned}$$

Here $\Delta G {\sim} G^{13} {\sim} G^{23}$, a measure of mountain steepness; $\Delta H {\sim} \Big(\frac{1}{G^{1/2}} - 1\Big)$, a measure of mountain height; $N^2 = \frac{g}{\theta}\dfrac{\partial\theta}{\partial z}$, the squared Brunt–Vaisala frequency; $\bar{U}$ and $\bar{V}$ are constant basic state wind velocity components; and $\theta'' = \frac{\rho'}{N}\frac{g\bar{\rho}}{\bar{\theta}}$.

The above two systems of equations, the original and linearized ones, are discretized by using numerical schemes with central differences in time and space, on grids for the scalar and vector quantities shifted half-grid size from each other in the space variables. The terms in the left-hand side of the linearized system are taken by central differences in time and space, while the terms in the right-hand side are taken at half-time grid levels (see, e.g., [9]).

We put $\bar{U}$, $\bar{V}$, and $\bar{N}$ equal to zero.

The total three-dimensional difference equations system may be written as follows:

$$(A + \Delta t C)S^{n+1} = (A - \Delta t C)S^{n-1} + BS^n,$$

where $S^n = n(P^n, U^n, V^n, W^n, \theta''^{\,n})'$, and the matrices $A$, $C$, and $B$ are as follows:

$$A = \begin{vmatrix} 1/C_2^s & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}, \quad C = \begin{vmatrix} 0 & ikx^* & iky^* & ikz^* & 0 \\ ikx^* & 0 & 0 & 0 & 0 \\ iky^* & 0 & 0 & 0 & 0 \\ ikz^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix},$$

$$B = 2\delta t \begin{vmatrix} 0 & -\Delta G ikz^{**} & -\Delta G ikz^{**} & -\Delta H ikz^* & 0 \\ -\Delta G ikz^{**} & 0 & 0 & 0 & 0 \\ -\Delta G ikz^{**} & 0 & 0 & 0 & 0 \\ -\Delta H ikz^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

where, in addition, $ky^* = \frac{2\sin(ky\frac{\Delta y}{2})}{\Delta y}$.

In [10], an eigenvalue problem for the amplification matrix was solved by using a procedure for matrices in Hessenberg form described by Wilkinson and Reinsch [11] (see also [12]). It was shown that, similar to a two-dimensional case considered in [9], a necessary stability limitation on $\Delta G$ is as follows:

$$0 \le \Delta G \le \gamma < 1,$$

where $\gamma$ is about 0.25.

## 4. Idealized front over a valley

In this paper, the front surface is treated by an efficient semi-Lagrangian finite difference scheme [13, 14]. Here the advection of a scalar is calculated in two steps:

Determination of the so-called departure point. This is the point from which the point under consideration is reached at the next time step.

Interpolation of values of the advected scalar from grid points on the departure point:

$$x_D = x - \int u\,dt, \qquad f(x, t + \Delta t) = f(x_D, t).$$

Here $u$ is the velocity vector and $\Delta t$ is the time step size. The scheme is designed as follows: an arbitrary function $f$ is expanded into a Taylor series up to terms of the fourth order. The free coefficients of this expansion are determined through values of the function at grid points. Denote $\lambda = (x_D - x_i)/\Delta x$. Here $\Delta x$ is the grid size. By solving the resulting system of linear equations for the free coefficients, we finally obtain:

$$f(t + \Delta t) = f_i(1 - \lambda/2 - \lambda^2 + \lambda^3/2) + f_{i+1}(\lambda + \lambda^2/2 - \lambda^3/2) + \\ f_{i+2}(-\lambda/6 - \lambda^2 + \lambda^3/6) + f_{i-1}(-\lambda/3 + \lambda^2/2 - \lambda^3/6).$$

A third-order semi-Lagrangian scheme was used as a reasonable compromise between computational complexity and accuracy.

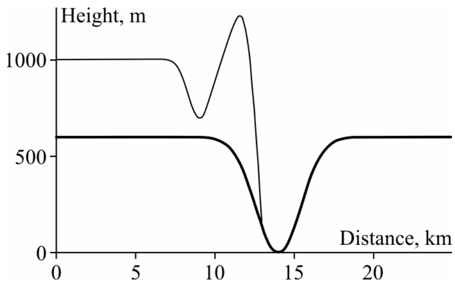


**Figure 1**

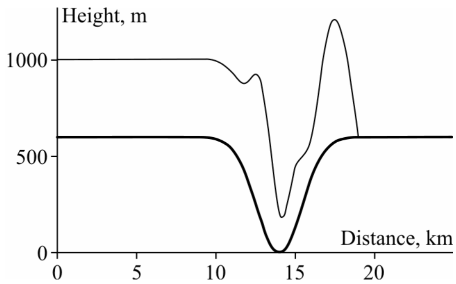


**Figure 2**

In this section, we apply the above constructions to simulating the propagation of an idealized cold atmospheric front over a valley in two dimensions. The input parameters are taken from [6]. The obstacle is a circular valley with an axially symmetric Gaussian shaped height profile of 600 m. The computational domain is $25 \times 2$ km. In contrast to [6], the front was not driven by a cold air source, but given initially as a step-function. Figure 1 shows the front as it enters the valley. In Figure 2, the front climbs the opposite side of the valley. A reasonable front propagation behavior is obtained, as compared to the results of [6].

## Conclusion

The results of calculations presented above are tentative. The study should be extended to more realistic situations usually described by sophisticated physical parameterizations. In forthcoming papers, the effects of stratification and valley shape on front propagation will be studied. Also, comparison will be made with simulation results on atmospheric front deformation by mountains and hills. Although the present study is of limited utility, the above simulation results show that the numerical tools proposed in this paper can be used for the numerical simulation of cold front propagation in the atmosphere.

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