

## Mean value theorem for a system of differential equations for the stress tensor and pore pressure\*

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**Abstract.** A system of second-order differential equations for the stress tensor and pore pressure for the poroelasticity statics without mass forces and energy dissipation is obtained. The stress tensor is shown to be a biharmonic function. Integral mean value relations in the explicit form for the obtained systems of differential equations are found.

### Introduction

It is well known that static simulation methods are used to solve multidimensional boundary value problems at a small number of points, especially, if the domain boundary shape is rather complex [1–3].

If a boundary value problem has stochastic parameters (for instance, the equation coefficients or the right-hand sides are random), Monte Carlo methods are an especially convenient tool to calculate both the average characteristics of the solution and other static characteristics [4].

Such theorems were proved for many basic equations and systems of equations (see [1–15]). In papers [16–18], systems of differential equations in terms of displacements of particles of an elastic porous body and pore pressure for stationary processes in the porous medium were obtained [19, 20]. Mean value relations for such systems of differential equations were also established [16–18].

In the present paper, a system of differential equations in terms of the stress tensor and pore pressure for stationary processes in the porous medium is obtained. Mean value relations for the obtained system of differential equations are found.

### 1. Problem statement

Assume that a bounded domain  $\Omega \subset R^3$  is a porous medium filled with a homogeneous isotropic saturated fluid. In the reversible case, the elastic-porous static state of the medium  $\Omega$  is described by the following system of differential equations [16–18]:

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$$\mu\Delta\mathbf{U} + (\tilde{\lambda} + \mu)\nabla\operatorname{div}\mathbf{U} = 0, \quad (1)$$

$$\Delta p = 0, \quad (2)$$

where  $\mathbf{U} = (U_1, U_2, U_3)$  is the displacement vector of an elastic porous body with the partial density  $\rho_s$ ,  $p$  is the pore pressure,  $\tilde{\lambda} = \lambda - (\rho^2\alpha)^{-1}K^2$ ,  $K = \lambda + 2\mu/3$ ,  $\lambda, \mu, \alpha = \rho\alpha_3 + K/\rho^2$  are the constants of the equation of state [20–22],  $\rho = \rho_s + \rho_l$ , and  $\rho_l$  is the partial density of the fluid. In [16], a formula was obtained relating the stress tensor with the deformation tensor of an elastic-porous body and pore pressure:

$$\sigma_{ik} = 2\mu\varepsilon_{ik} + \tilde{\lambda}\delta_{ik}\bar{\varepsilon} - \hat{\alpha}\delta_{ik}p, \quad i, k = 1, 2, 3, \quad (3)$$

$$\varepsilon_{ik} = \frac{1}{2}(U_{i,k} + U_{k,i}), \quad \bar{\varepsilon} = \sum_{n=1}^3 \varepsilon_{nn},$$

where  $\delta_{ik}$  is the Kronecker symbol,  $v_{,k} = \frac{\partial v}{\partial x_k}$ ,  $\hat{\alpha} = 1 - \frac{K}{\alpha\rho^2}$ .

Solving system (3) with respect to the deformation tensor, we obtain

$$\varepsilon_{ik} = \frac{1}{2\mu}\sigma_{ik} - \frac{\delta_{ik}}{3\tilde{\lambda} + 2\mu} \left( \frac{\tilde{\lambda}}{2\mu}\bar{\sigma} - \hat{\alpha}p \right), \quad i, k = 1, 2, 3. \quad (4)$$

## 2. System of differential equations for the stress tensor and pore pressure

Substituting (3) into the consistency condition of the deformation tensor

$$\varepsilon_{ij,kk} + \varepsilon_{kk,ij} = \varepsilon_{ik,kj} + \varepsilon_{kj,ik}, \quad i, j, k = 1, 2, 3,$$

we obtain the following system of second order differential equations for the stress tensor and pore pressure:

$$\begin{aligned} & \sigma_{ij,kk} + \sigma_{kk,ij} - \frac{1}{3\tilde{\lambda} + 2\mu} \left[ \tilde{\lambda}\delta_{ij}\bar{\sigma}_{,kk} + \tilde{\lambda}\delta_{kk}\bar{\sigma}_{,ij} - 2\mu\hat{\alpha}\delta_{ij}p_{,kk} - 2\mu\hat{\alpha}\delta_{kk}p_{,ij} \right] \\ & = \sigma_{ik,kj} + \sigma_{kj,ik} - \frac{1}{3\tilde{\lambda} + 2\mu} \left[ \tilde{\lambda}\delta_{ik}\bar{\sigma}_{,kj} + \tilde{\lambda}\delta_{kj}\bar{\sigma}_{,ik} - 2\mu\hat{\alpha}\delta_{ik}p_{,kj} - 2\mu\hat{\alpha}\delta_{kj}p_{,ik} \right]. \end{aligned}$$

Let us perform summation over  $k$ :

$$\begin{aligned} & \Delta\sigma_{ij} + \bar{\sigma}_{,ij} - \frac{1}{3\tilde{\lambda} + 2\mu} \left[ \tilde{\lambda}\delta_{ij}\Delta\bar{\sigma} + 3\tilde{\lambda}\bar{\sigma}_{,ij} - 2\mu\hat{\alpha}\delta_{ij}\Delta p - 6\mu\hat{\alpha}p_{,ij} \right] \\ & = \sum_{k=1}^3 (\sigma_{ik,kj} + \sigma_{kj,ik}) - \frac{2}{3\tilde{\lambda} + 2\mu} \left[ \tilde{\lambda}\bar{\sigma}_{,ij} - 2\mu\hat{\alpha}p_{,ij} \right], \quad i, j = 1, 2, 3. \quad (5) \end{aligned}$$

From equation (1), with allowance for (3), for  $\sigma_{ik}$  we obtain the first order equation

$$\sum_{k=1}^3 \sigma_{ik,k} + \hat{\alpha} p_{,i} = 0, \quad i = 1, 2, 3. \quad (6)$$

From (5), with harmonicity of the pore pressure  $p$  and the equilibrium equation (6), we obtain

$$\Delta \sigma_{ij} + \frac{2(\tilde{\lambda} + \mu)}{3\tilde{\lambda} + 2\mu} \bar{\sigma}_{,ij} - \frac{\tilde{\lambda} \delta_{ij}}{3\tilde{\lambda} + 2\mu} \Delta \bar{\sigma} = 2\hat{\alpha} \frac{3\tilde{\lambda} + \mu}{3\tilde{\lambda} + 2\mu} p_{,ij}, \quad i, j = 1, 2, 3. \quad (7)$$

Hence, at  $i = j$  and summing over  $i$  from 1 to 3, we obtain harmonicity of the stress tensor trace  $\sigma_{ik}$ , that is,

$$\Delta \bar{\sigma} = 0. \quad (8)$$

With allowance for this equality, relation (7) takes the following form:

$$\begin{aligned} \Delta \sigma_{ij} + \beta \bar{\sigma}_{,ij} &= \gamma p_{,ij}, \quad i, j = 1, 2, 3, \\ \beta &= \frac{2(\tilde{\lambda} + \mu)}{3\tilde{\lambda} + 2\mu}, \quad \gamma = 2\hat{\alpha} \frac{3\tilde{\lambda} + \mu}{3\tilde{\lambda} + 2\mu}. \end{aligned} \quad (9)$$

Thus, the pore pressure and stress tensor satisfy the system of second order differential equations (2) and (9). It follows from system (9) that the stress tensor components are biharmonic functions. In fact, let the Laplace operator  $\Delta$  act on both sides of equality (9), and, taking into account properties (2) and (8), we obtain  $\Delta^2 \sigma_{ij} = 0$ .

### 3. The mean value relation for system (2), (9)

Now, according to [9], we introduce  $N(\mathbf{u})$ , which is the averaging operator of the vector function  $\mathbf{u} = (u_1, u_2, \dots, u_n)^T$  over the surface of a sphere  $S(\mathbf{x}, R)$  with respect to the uniform measure  $dS$ , that is,

$$N(\mathbf{u}) = \frac{1}{\omega_n r^{n-1}} \int \mathbf{u}(\mathbf{x} + r\mathbf{y}) dS(\mathbf{y}),$$

where  $\omega_n$  is the unit sphere area, and  $\{s_i\}_{i=1}^n$  are the direction cosines.

For the harmonic function  $p(\mathbf{x})$ ,  $\mathbf{x} \in \Omega$ , the mean value relations [10]

$$p(0) = \frac{\int_{S(0,R)} p d\Omega}{\int_{S(0,R)} d\Omega} = \frac{1}{\omega_3} \int_{S(0,1)} p dS, \quad (10)$$

$$p(0) = \frac{3}{4\pi R^3} N^{(W)} p(\mathbf{x}), \quad (11)$$

are valid. Here  $N^{(W)} p(\mathbf{x})$  is the integral of  $P$  over the ball  $W(\mathbf{x}, R) = \{|\mathbf{x} - \mathbf{y}| < R\}$ . For the harmonic function  $\frac{\partial^2 p}{\partial x_k \partial x_i}$ ,  $i, k = 1, 2, 3$ , we use relation (11):

$$\frac{\partial^2 p(0)}{\partial x_k \partial x_i} = \frac{3}{4\pi R^3} \int_{W(0,R)} p_{,ik} dW = \frac{3}{4\pi R^3} \int_{S(0,1)} p \frac{x_k x_i}{R^2} dS.$$

From now on,  $W_t := W(0, t)$ ,  $S_t := S(0, t)$ .

As shown in [10], the mean value relation is valid for a biharmonic function. Applying formula (2.5) from [10] to equation (6), we obtain

$$\sigma_{ij}(0) = \frac{3}{2\omega_3} \left[ \frac{5}{R^3} \int_{W_R} \sigma_{ij} dW - \int_{S_1} \sigma_{ij} dS \right] \quad (12)$$

From the equilibrium equation (6), we have

$$\begin{aligned} \int_{W_\eta} \sigma_{ij} dW &= \int_{W_\eta} (\sigma_{ik} x_j)_{,k} dW - \int_{W_\eta} \sigma_{ik,k} x_j dW \\ &= \eta^3 \int_{S_1} \sigma_{ik} \frac{x_j x_k}{\eta^2} dS + \hat{\alpha} \int_{W_\eta} (p x_j)_{,i} dW - \hat{\alpha} \delta_{ij} \int_{W_\eta} p dW \\ &= \eta^3 \int_{S_1} \sigma_{ik} \frac{x_j x_k}{\eta^2} dS + \hat{\alpha} \eta^3 \int_{S_1} p \frac{x_i x_j}{\eta^2} dS - \hat{\alpha} \delta_{ij} \int_{W_\eta} p dW. \end{aligned} \quad (13)$$

From (12) and (13), we obtain

$$\begin{aligned} \sigma_{ij}(0) &= \frac{3}{2\omega_3} \left[ 5 \int_{S_1} \sigma_{ik} \frac{x_j x_k}{\eta^2} dS - \int_{S_1} \sigma_{ij} dS + \right. \\ &\quad \left. 5\hat{\alpha} \int_{S_1} p \frac{x_i x_j}{\eta^2} dS - \frac{5\hat{\alpha}}{\eta^3} \delta_{ij} \int_{W_\eta} p dW \right]. \end{aligned}$$

Now we multiply both sides of this equality by  $\eta^4$ , integrate from 0 to R and, with allowance for (11), obtain

$$\begin{aligned} \frac{R^5}{5} \sigma_{ij}(0) &= \frac{3}{2\omega_3} \left[ 5 \int_{S_1} \sigma_{ik} \frac{x_j x_k}{\eta^2} dS - \int_{S_1} \sigma_{ij} dS + \right. \\ &\quad \left. 5\hat{\alpha} \int_{S_1} p \frac{x_i x_j}{\eta^2} dS \right] - \frac{R^5 \hat{\alpha}}{2} \delta_{ij} p(0). \end{aligned} \quad (14)$$

From the equilibrium equation (6), as in (13), we obtain

$$\begin{aligned} \int_{W_R} \eta^2 \sigma_{ij} dW &= R^5 \int_{S_1} \sigma_{ik} \frac{x_j x_k}{R^2} dS - 2 \int_{W_R} \sigma_{ik} x_k x_j dW + \\ &\quad \hat{\alpha} R^5 \int_{S_1} p \frac{x_i x_j}{R^2} dS - \hat{\alpha} \delta_{ij} \int_{W_R} \eta^2 p dW - 2\hat{\alpha} \int_{W_R} p x_i x_j dW. \end{aligned} \quad (15)$$

Let us multiply (9) by  $\eta^2$  and integrate with respect to the ball,

$$\begin{aligned}
 0 &= \int_{W_\zeta} \eta^2 [\Delta \sigma_{ij} + \beta \bar{\sigma}_{,ij} - \gamma p_{,ij}] dW \\
 &= \zeta^2 \int_{S_\zeta} \left[ \sigma_{ij,k} \frac{x_k}{\zeta} + \beta \bar{\sigma}_{,i} \frac{x_j}{\zeta} - \gamma p_{,i} \frac{x_j}{\zeta} \right] dS - \\
 &\quad 2 \int_{W_\zeta} \left[ \sigma_{ij,k} \frac{x_k}{\zeta} + \beta \bar{\sigma}_{,i} \frac{x_j}{\zeta} - \gamma p_{,i} \frac{x_j}{\zeta} \right] dW \\
 &= -2\zeta^3 \int_{S_1} \left[ \sigma_{ij} + \beta \bar{\sigma} \frac{x_i x_j}{\zeta^2} - \gamma p \frac{x_i x_j}{\zeta^2} \right] dS - \\
 &\quad \int_{W_\zeta} (3\sigma_{ij} + \beta \delta_{ij} \bar{\sigma} - \gamma \delta_{ij} p) dW. \tag{16}
 \end{aligned}$$

Here we make use of the fact that the surface integral is zero and the Gaussian formula.

Assuming in (13) that  $i = j = k$ , we obtain

$$\int_{W_\eta} \sigma_{kk} dW = \eta^3 \int_{S_1} \sigma_{kl} \frac{x_k x_l}{\eta^2} dS + \hat{\alpha} \eta^3 \int_{S_1} p dS - 3\hat{\alpha} \int_{W_\eta} p dW. \tag{17}$$

From (16) and (17), using simple transformations, we have

$$\begin{aligned}
 &3 \int_{W_R} \sigma_{ik} x_j x_k dW + 3\hat{\alpha} \int_{W_R} p x_i x_j dW - \\
 &\quad \left( \hat{\alpha} + \frac{\gamma}{3} \right) \delta_{ij} \int_{W_R} \eta^2 p dW + \beta \delta_{ij} \int_{W_R} \sigma_{kl} x_k x_l dW \\
 &= \int_{W_R} \eta^2 \sigma_{ij} dW + \beta \int_{W_R} \bar{\sigma} x_i x_j dW - \gamma \int_{W_R} p x_i x_j dW. \tag{18}
 \end{aligned}$$

It can be shown by direct calculations that  $\sigma_{kl} x_k x_l$  is a biharmonic function. Using for it formula (12), we obtain

$$0 = \frac{5}{R^3} \int_{W_R} \sigma_{kl} x_k x_l dW - \int_{S_1} \sigma_{kl} x_k x_l dS. \tag{19}$$

From (18), with allowance for (15) and (19), we have

$$\begin{aligned}
 5 \int_{W_R} \sigma_{ik} x_j x_k dW &= \beta \int_{W_R} \bar{\sigma} x_i x_j dW + \\
 &\quad R^5 \left[ \int_{S_1} \sigma_{ik} \frac{x_j x_k}{R^2} dS - \frac{\beta}{5} \delta_{ij} \int_{S(0,1)} \sigma_{kl} \frac{x_k x_l}{R^2} dS \right] - \\
 &\quad (5\hat{\alpha} + \gamma) \int_{W_R} p x_i x_j dW - \left( \hat{\alpha} + \frac{\gamma}{3} \right) \delta_{ij} \int_{W_R} \eta^2 p dW + \\
 &\quad \frac{\gamma}{3} \delta_{ij} \int_{W_R} \eta^2 p dW + \hat{\alpha} R^5 \int_{S_1} p \frac{x_i x_j}{R^2} dS \tag{20}
 \end{aligned}$$

The volume integral in (20) can be transformed as follows:

$$\begin{aligned}
\int_{W_R} \sigma_{kk} x_i x_j dW &= \int_{W_R} (\sigma_{kl} x_m x_i x_j)_{,l} dW - \int_{W_R} \sigma_{kl,l} x_k x_i x_j dW - \\
&\int_{W_R} \sigma_{ki} x_k x_j dW - \int_{W_R} \sigma_{kj} x_k x_i dW \\
&= R^5 \int_{S_1} \sigma_{kl} \frac{x_i x_j x_k x_l}{R^4} dS + \hat{\alpha} R^5 \int_{S_1} p \frac{x_i x_j}{r^2} dS - \\
&3\hat{\alpha} \int_{W_R} p x_i x_j dW - \int_{W_R} (\sigma_{ki} x_k x_j + \sigma_{kj} x_k x_i) dW. \quad (21)
\end{aligned}$$

In the derivation of (21), we used the equilibrium equation (6).

From (20) and (21), after simple transformations, we obtain

$$\begin{aligned}
\int_{W_R} \sigma_{ik} x_j x_k dW &= \frac{R^5}{5+2\beta} \left[ \beta \int_{S_1} \sigma_{kl} \frac{x_i x_j x_k x_l}{R^4} dS + \int_{S_1} \sigma_{ik} \frac{x_j x_k}{R^2} dS - \right. \\
&\left. \frac{\beta}{5} \delta_{ij} \int_{S_1} \sigma_{kl} \frac{x_k x_l}{R^2} dS \right] + \frac{\hat{\alpha}(\beta+1)}{5+2\beta} R^5 \int_{S_1} p \frac{x_i x_j}{R^2} dS + \\
&\frac{\gamma}{3(5+2\beta)} \delta_{ij} \int_{S_1} \eta^2 p \frac{x_i x_j}{R^2} dW - \frac{(5+3\beta)\hat{\alpha} + \gamma}{5+2\beta} \int_{W_R} p x_i x_j dW. \quad (22)
\end{aligned}$$

From (16) and (22), after simple transformations, we obtain the integral mean value relations theorem for the stress tensor:

**Theorem.** *For the system of differential equations (9), the following mean value relations are valid:*

$$\begin{aligned}
\sigma_{ij}(0) &= \frac{3}{2\omega_3(5+2\beta)} \left[ 10(1-\beta) \int_{S_1} \sigma_{ik} \frac{x_k x_j}{R^2} dS - 7\beta \delta_{ij} \int_{S_1} \sigma_{kl} \frac{x_k x_l}{R^2} dS + \right. \\
&35\beta \int_{S_1} \sigma_{kl} \frac{x_i x_j x_k x_l}{R^4} dS \left. \right] + \frac{15\hat{\alpha}(2+5\beta)}{2\omega_3(5+2\beta)} \int_{S_1} p \frac{x_i x_j}{R^2} dS + \\
&\frac{15}{2\omega_3 R^5} \left( \hat{\alpha} + \frac{7\gamma}{3(5+2\beta)} \right) \delta_{ij} \int_{W_R} \eta^2 p dW + \\
&\frac{105(\hat{\alpha}\beta + \gamma)}{2\omega_3(5+2\beta)R^5} \int_{W_R} p x_i x_j dW - \frac{5\hat{\alpha}}{2} \delta_{ij} p(0). \quad (23)
\end{aligned}$$

Thus, we have obtained integral mean value relations (7), (8), and (23) for a system of poroelasticity equations. To determine the dilatancy zone, it is necessary to have integral characteristics of the medium being considered. In mathematical simulation, the averaging method—the mean value theorem is used for this. The relations obtained for the stress tensor of a porous body and pore pressure allow using dilatancy zones in problems of monitoring the Earth's crust technogenic processes and earthquake prediction [23].

Since in (23)  $\hat{\alpha}$  tends to zero, we obtain mean value relations for the stress tensor components for the static equations of classical elasticity [10].

**References**

- [1] Dyad'kin I.G., Starikov V.N. A possible way of economizing computer time in the solution of the Laplace equation by the Monte Carlo method // *Zh. Vych. Mat. i Mat. Fiziki.* — 1965. — Vol. 5, No. 5. — P. 936–938.
- [2] Elepov B.S. et al. *Solution of Boundary Value Problems by the Monte-Carlo Method.* — Novosibirsk: Nauka, 1980.
- [3] Sobol' I.M. *Monte Carlo Numerical Methods.* — Moscow: Nauka, 1973.
- [4] Sabelfeld K.K. *Monte Carlo Methods in Boundary Value Problems.* — Novosibirsk: Nauka, 1989.
- [5] Courant R. *Solving Boundary Value Problems by the Monte Carlo Method.* — Novosibirsk: Nauka, 1980.
- [6] Aquaro G. Un teorema di media per le equazioni dell'elasticità // *Riv. Mat. Univ. Parma.* — 1950. — P. 419–424.
- [7] Synge J.L. Upper and lower bounds for solutions of problems in elasticity // *Proc. Roy. Irish Acad.* — 1950. — Vol. 53. — P. 41–64.
- [8] Ermakov S.M., Nekrutkin V.V., Sipin A.S. *Random Processes for Classical Equations of Mathematical Physics.* — Moscow: Nauka, 1984.
- [9] Sabelfeld K.K., Shalimova I.A. *Spherical Means for PDEs.* — Utrecht: VSP, 1997.
- [10] Diaz J.B., Payne L.E. On a mean value theorem, and its converse, for the displacements in the theory of elasticity // *Portugaliae Mathematica.* — 1958. — Vol. 17, No. 4. — P. 123–126.
- [11] Bramble J.H., Payne L.E. Some converses of mean value theorems in the theory of elasticity // *J. Math. Anal. Appl.* — 1965. — Vol. 10, No. 3. — P. 553–567.
- [12] Bitsadze A.V. *Some Classes of Partial Differential Equations.* — Moscow: Nauka, 1981.
- [13] Naumov V.V. Mean value theorems for the equation of harmonic oscillations of an elastic body // *Fluid Dynamics: Coll. Sci. Proc. USSR Acad. Sci. Sib. Branch. Institute of Hydrodynamics.* — 1987. — Iss. 82. — P. 147–153.
- [14] Grigor'ev Yu.M. Mean value theorems for heterogeneous Helmholtz and Lamé equations // *Fluid Dynamics: Coll. Sci. Proc. USSR Acad. Sci. SB RAS. Institute of Hydrodynamics.* — 1988. — Iss. 113. — P. 53–59.
- [15] Grigor'ev Yu.M., Naumov V.V. On mean value theorems for the Helmholtz and Lamé equations // *Dokl. Ross. Akad. Nauk.* — 1998. — Vol. 362, No. 1. — P. 51–52.

- [16] Grachev E.V., Zhabborov N.M., Imomnazarov Kh.Kh. A concentrated force in an elastic porous half-space // *Dokl. Ross. Akad. Nauk.* — 2003. — Vol. 391, No. 3. — P. 331–333.
- [17] Grachev E., Imomnazarov Kh., Zhabborov N. One nonclassical problem for the statics equations of elastic-deformed porous media in a half-plane // *Applied Mathematics Letters.* — 2004. — Vol. 17, Iss. 1. — P. 31–34.
- [18] Zhabborov N.M., Imomnazarov Kh.Kh. Theorem on spherical mean for a static poroelasticity system // *Proc. Intern. Conf. “New Trends in the Theory of Dynamic Systems and Ill-Posed Problems”*, Samarkand. — 2007. — P. 261–263.
- [19] Bonnet G. Basic singular solutions for a poroelastic medium in the dynamic range // *J. Acoust. Soc. Am.* — 1987. — Vol. 82. — P. 1758–1762.
- [20] Gorog S., Panneton R., Atalla N. Mixed displacement-pressure formulation for acoustic anisotropic open porous media // *J. Appl. Phys.* — 1997. — Vol. 82. — P. 4192–4196.
- [21] Dorovsky V.N. Continual theory of filtration // *Geologiya i Geofisika.* — 1989. — No. 7. — P. 39–45.
- [22] Dorovsky V.N., Perepechko Yu.V., Romensky E.I. Wave processes in saturated porous elastically deformed media // *Combustion, Explosion and Shock Waves.* — 1993. — No. 1. — P. 100–111.
- [23] Blokhin A.M., Dorovsky V.N. *Mathematical Modelling in the Theory of Multivelocity Continuum.* — Nova Science Publishers, Inc., 1995.
- [24] Mikhailenko B.G., Glinsky, B.M., Imomnazarov Kh.Kh., et al. Active seismology methods in problems of Earth’s deep structure monitoring // *Extreme Natural Phenomena and Catastrophes.* — Vol. 1: Estimation and Ways to Decrease Negative Consequences of Extreme Natural Phenomena / *Man. Ed.:* A.O. Gliko; *Man. Comp. A.L. Sobisevich.* — Moscow: Institute of Physics of the Earth RAS, 2010. — P. 89–130.